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RELIABILITY
HANDBOOK

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UNITED STATES ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
PRODUCT ASSURANCE DIRECTORATE
DOVER, NJ 07801

RELIABILITY HANDBOOK

Second Edition

July 1979

NUCLEAR SYSTEMS DIVISION

PRODUCT ASSURANCE DIRECTORATE

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND

DOVER, NEW JERSEY

See also
AD-457,170

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PREFACE

This handbook is intended as a guide for determining reliability of functioning characteristics of weapon components by testing to failure.

Component reliability of weapon systems is basically a function of engineering design. Margins of safety used in engineering design to create high reliabilities must be measured by testing to failure techniques to obtain unbiased estimates of reliability.

The author does not hold that the concepts and principles presented herein are final. Revisions will inevitably be made as the state of the art advances.

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SUMMARY

1. The following are set forth:

    a. The concept of reliability of functioning characteristics of weapon components in terms of stress and strength.

    b. The operating engineering definition of reliability.

    c. The complex nature of reliability.

    d) Ultimate reliability in terms of safety margins.

    e. The relationship between test and use conditions.

    f) The limitations of reliability determinations imposed by testing facilities, information and cost.

2. A *two-phase testing procedure* which meets the need for demonstrating *high reliabilities with small sample sizes* is described in a rational, objective manner. The first phase involves use of *fractional-factorial experimental designs* to survey effects of important environments. The second phase is a *test-to-failure* procedure (using the *environment found most severe* in the *first phase*) so conducted that reliability-in-use can be calculated from test results.

3. The need to plan experiments in advance of data collection and, to test to failure and emphasized. The requirements of a good experiment are treated.

4. Several useful fractional-factorial test plans are completely laid out in the form of treatment procedures. Tests of increased severity most useful in testing to failure are described. Examples are given for applying these methods.

5. Useful statistical tables, a glossary of terms, and a list of references are included.

INTRODUCTION

This handbook has been prepared for those engineers and scientists conducting reliability experiments who would like to use statistical techniques to *improve the efficiency of their experiments*. However, it is advisable, especially in the planning stages of testing programs, to supplement the information in this manual by occasional consultations with a statistician.

Planning experiments in the modern statistical sense compels the experimenter explicitly to formulate his objectives and the procedures required to attain them. This often leads to the recognition of fallacies and other difficulties in advance of data collecting.

The statistical aspects of reliability are not new. All of the necessary concepts are adequately treated in modern statistical literature. The *lack of information about measurable characteristics of the missile system and the environment it experiences in use*, as well as the *high cost of test specimens, have created the current problems*.

The techniques described in this manual are the most efficient known. They are designed to *maximize the amount of information obtainable from a given sample size. Very high reliabilities* (0.9999 and higher if they exist) *can be demonstrated from very small sample sizes (25 to 30 items)*. In addition, these techniques are definitive enough to serve as standard procedures throughout the same or different organizations over extended periods of time.

Uniform application of these techniques is as important as their efficiency. A large part of the value of experimentally determined reliability data is the scope of applicability. Reliability data collected by means of standardized procedures are cumulative in the mathematical sense. Hence, the *precision with which reliability values are known can be improved with time as additional data are collected. This makes it possible to accumulate a reference file of reliability data on a variety of standard components*.

For those readers not thoroughly familiar with statistical terms a glossary of these terms has been included in Appendix 2.

1. ## RELIABILITY CONCEPT

It is assumed that for every missile component there exists a true but unknown "strength" created by the particular (parts) design developed and used by the engineer in building the component. It is further assumed that the true "strength" is a constant and not a random variable for any particular design over short periods of time.

An item will not fail until the applied stress exceeds the items "strength." If the "strength" is much greater than the stress expected to be experienced in use, the chance (probability) of failure in use is very small, and the chance of success (reliability) is very high. It is in this sense that "high reliability" is defined. That is, high reliability means high probability of successful functioning under actual *use* conditions; it does not mean high reliability under the *test* conditions.

RELIABILILITY DEFINITION

The accepted statistical definition of reliability is that reliability is "the probability of successful functioning in use." This is a general definition that is applicable to any operating system. However, to define reliability from an operating engineering point of view, the general definition must be modified to include:

a. The environmental conditions under which successful functioning is required.

b. The characteristics that are required to function successfully.

c. The length of time or the number of times successful functioning is required.

d. When successful functioning is required.

This means that every component can have as many reliabilities as a number equal to the possible combinations of environmental conditions, measurable characteristics and functioning times.

Under the definition that an item cannot fail until the stress exceeds its strength, the reliability with respect to any environment can be determined only if the test specimens are stressed by that environment until failure is obtained. This means that successively higher levels of severity must be used until failure is obtained, as in the applications of successively greater loads until failure is obtained in a tensile test. When the magnitude of the stress at the point of failure cannot be directly observed, then an exploratory type test such as the Bruceton up-and-down method is required. This procedure generates the failure rate curve from which the average (ultimate) strength (the point at which the stress equals the strength) can be obtained by finding the stress at which 50 percent of the items fail. In any case the ultimate strength of an item is determined in such a manner that the reliability-in-use can be predicted from the test results.

3. # LIMITATIONS

A. Laboratory Testing

In laboratory testing, it is difficult to reproduce the conditions which components will experience operationally. In use, several environments occur simultaneously; in the laboratory, the environments usually have to be applied in sequence. As a result, the environment experienced in use is more severe than that applied at comparable levels of severity in the laboratory. Furthermore, interactions among environments and among components raise the level of severity experienced in use by an additional amount. The extent to which the level of severity is increased in these cases is usually not known.

To cope with unknowns of this kind, engineers use "margins (or factors) of safety" to assure successful functioning in use. As a rational consequence, testing procedures used to test components must, to be of any value, determine the actual margins of safety the engineers have succeeded in building into the new item. To accomplish this, with the limitations imposed by cost considerations, careful planning prior to data collection is required. Useful and realistic component reliability values cannot be obtained by accident or as a by-product of a testing program designed for some other purpose, such as controlling quality. However, reliability values can supplement but not replace quality control and other engineering information.

B. Information

A complicated system of any kind cannot be fully characterized or described by a single numerical value. Just as the "whole man" cannot be fully described by an intelligence quotient, a whole missile system cannot be fully described or characterized by a single reliability value. Fully to characterize the expected performance of a missile, all possible reliabilities should be:

    a. Determined and weighted in accordance with:

        (1) Their engineering importance,

        (2) Probability of occurrence of the various environments,

        (3) Duration and intensity of the environment,

        (4) Presence of interaction among environments and among components, and

    b. Mathematically combined:

        (1) In accordance with the way the environments occur (i.e., simultaneously, in combination, or in sequence),

        (2) In various ways to predict the probability of successful functioning of the major and minor subassemblies,

        (3) In accordance with the system circuitry to predict the reliabilities of the over-all system.

C. Cost

The cost of measuring the magnitude and interaction effects of the multitude of variables affecting performance of complex missile systems is prohibitive, as in the cost of determining *all* of an item's possible reliability values, or even a large number of these values. These costs will perhaps remain prohibitive as long as there is a reasonable alternative.

4. SAFETY MARGINS

The use of safety margins to assure successful functioning under unpredictable conditions is not new. Currently, reliance is placed on the "safety factor" or the "margin of safety" as an alternative for information. If the expected nominal "stress" (or load) in use is 100 units, designing an item with the "strength" to withstand several times this "stress" gives intuitive assurance that the item will function successfully without failure (i.e., be reliable). Such an item will surely withstand 100 units of "stress" (be highly reliable under this condition) and has a good chance of functioning successfully even when the applied "stress" varies widely, the quality of the material is substandard, or the workmanship is poor. A large margin of safety, then, is a means of assuring successful functioning in the presence of uncontrolled and indeterminate variations in environment, materials, and workmanship. This concept of "stress" and "strength" can be used as a corollary to the definition of reliability given above: An item cannot fail until the stress exceeds its strength. The point at which the stress equals the strength measures the average (ultimate) strength. At this point the reliability equals 50 percent. To raise the reliability above this level, the strength must exceed the expected in-use stress. High strength relative to the stress means high reliability, since the higher the strength, the less likely a failure is to occur.

Construction engineers design an item to withstand several times the load expected in use (for the above reasons), then evaluate the design by measuring the safety factor of a few representative specimens. This can only be done by applying a load until the specimens break, or fail in some other manner. The breaking load is a measure of the ultimate strength. The *safety factor* is the ultimate strength divided by the load expected in use. The *margin of safety* is the difference between those two loads divided by the expected load in use. Calculating either of these values is as far as construction engineers usually go; they do not calculate a numerical value for the probability of success in use (reliability) created by the safety factor. If the safety factor is large, they *feel* confident in concluding (predicting) the item will not fail in use.

Missile engineers also use safety margins. They design margins of safety into missile components in many subtle ways and for the same reasons: to assure successful performance in use under uncontrolled and unpredictable conditions. Here, too, the "margin of safety" designed into an item can only be determined by testing to failure. The "stress" required to cause failure can also be termed ultimate "strength."

5. ULTIMATE RELIABILITY

The reliability obtained by *testing to failure* is the ultimate (maximum) reliability, whether a margin of safety is used or not. This is the only *unbiased* measure of the true reliability created by the design of the item.

Testing *without* failure demonstrates reliability only in proportion to the number of test specimens used. This is a *biased* estimate of the ultimate reliability. This means that the ultimate reliability cannot be determined by testing a finite number of specimens without failure.

6. ## PLANNED EXPERIMENTS

When only one of the possible total number of reliabilities can be determined, the logical choice is to determine the minimum reliability. If the latter is satisfactory, all other possible reliabilities with respect to separate environments will also be satisfactory. Without a knowledge of the values of *all* reliabilities, meaningful and realistic system reliabilities can only be predicted from component reliabilities on the basis of the minimum reliabilities.

Experiments must be *designed**. This requires planning in advance of data collection. Test plans must be specifically designed to assure, in advance of data collection, that specified objectives will be attained, for reliability can neither be *tested*, nor *analyzed into* an item.

Environmental conditions which cause the poorest (minimum) reliabilities can be found most efficiently (with smallest sample sizes) by means of fractional-factorial designs or their optimized modifications. The object here is to survey environments considered most important to the functioning of the item and to find the environment having the most severe effect (i.e., causing the lowest reliability). This environment is then used to determine the minimum ultimate reliability by testing to failure, using tests of increased severity.

7. ## TESTING WITHOUT FAILURE

The margin of safety designed into a missile component can be determined only by testing to failure. To do otherwise, practically nullifies the value of test results and makes the engineer's effort to use safety margins ineffective. If the test procedure does *not* measure safety margins, the engineer has no evidence that they exist, and may conclude that other means of increasing reliability (e.g., redundancy) must be used. This line of action is not only costly, bu may create other problems, such as: misplaced center of gravity, overweight, and lack of space.

Testing without failure, which entails large sample sizes, is costly. By this method, it takes 460 items to demonstrate a reliability of at least 99.5 percent with 90 percent confidence. The same reliability can be demonstrated (if it exists in the item) with 25-30 items by testing to failure, using tests of increased severity. In addition the results of testing without failure causes difficulty in calculating system reliability from component reliability because zeros cannot be mathematically manipulated.

Because testing without failure cannot measure ultimate reliability with small sample sizes, trends cannot be detected early enough for taking timely corrective action. For example, if the ultimate reliability of an item actually exceeds that specified in the military characteristics (0.995 or higher), and only 25 items are tested at the use condition without failure during each testing period, no trend will be detected until ultimate reliability of the lot, or stockpile, drops below 0.91 (at the 90 percent confidence level).

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* See Reference 12

8.   RELATION BETWEEN TEST AND USE CONDITIONS

To translate the reliability demonstrated under test conditions to a "reliability-in-use" value, the relation between the "use" and "test" conditions must be established. Experience has shown that this relationship can be adequately represented by frequency distributions. This places the relationship on a probabilistic basis, and also makes possible the use of the laws of probability. If then, the test results are properly collected (see Lab Test Methods), the reliability-in-use can be calculated by extrapolation.

9.   TEST PLANS

Plans should be made to conduct experiments in two stages:

A. Factorial experiments:

For each type of component, the separate effects of the critical environments can be determined most efficiently in one integrated factorial experiment. From these results, the environments having the most severe effects should be selected. When only attribute data can be obtained the optimum condition for conducting this experiment is at a level of severity at which approximately 50 percent of the test specimens can be expected to fail. This type of experiment is highly efficient. The effect of as many as 7 environments can be determined with 8 test specimens, or the effects of 15 environments with 16 test specimens.

B. Testing to Failure

Within the limitations imposed upon the experiment, determine the reliability with respect to as many as possible of the environments having the most severe effects. This can be accomplished most efficiently using a test of increased severity such as the Bruceton up-and-down method. It is only with this type of test that the ultimate "strength" can be determined when the occurrence of a failure cannot be detected by inspection or when the magnitude of the stress at the point of failure cannot be directly observed. From this information the predicted ultimate "reliability-in-use" can be calculated. Ultimate "reliability-in-use" of any magnitude that exists in an item can be demonstrated with as few as 25 to 30 test specimens by testing the failure with tests of increased severity.

MODERN STATISTICAL CONCEPTS

1. INTRODUCTION

Because of the nature of reliability and because of the methods required to determine reliability, modern statistical concepts, such as probability, experimental error, population — sample relation, frequency distributions, confidence intervals, sample size, and design of experiment techniques must be understood, if reliability experiments are to be conducted and reliability values calculated and interpreted. It is only with these concepts that the vexing problem of demonstrating high reliabilities with small sample sizes can be solved.

Modern statistical methods of experimentation contain a new ingredient not explicit in mathematics: *Error.* The new philosophy assumes that there is an error in every measurement made and as a consequence, the true values of measurable characteristics can never be known exactly. To cope with this deficiency of measuring processes, repeated measurements are made. Then from this data an interval is calculated which we believe includes the true value represented by the data. Intervals of this kind are called confidence intervals.

Included in the method of calculating these intervals is a means of controlling the proportion of the time that the true value is expected to fall within the interval. Thus the name. This proportion expresses our "confidence" of being right in our prediction that the true value will fall in the interval calculated. Formulas for calculating confidence intervals are given below in Section IX: Reliability Confidence Intervals.

2. EXPERIMENTAL ERRORS

If the same characteristic is repeatedly measured with an "accurate" device under constant conditions, the same result will not always be obtained. As a matter of fact, the same result will seldom be repeated. However, it will be noticed that most of the values will cluster rather closely. Only a few very small and very large values will be obtained. It is assumed that these observed deviations are due to chance errors in the measuring process. They are called experimental errors.

3. POPULATION VS SAMPLE

The family of values generated by repeated measurements of the same characteristic is called a *population.* A population is generally assumed to be infinite. Any sub-portion of a population is called a sample of that population. A sample is always finite.

4. PREDICTION ERRORS

The reasoning behind the new philosophy is as follows: The observations or measurements made in any experiment are, in fact, finite samples of a much larger (infinite) body of data that

could exist had thousands (infinite) of observations been made of the same characteristic under the same constant conditions. It is assumed that unless an infinite number of observations is made, the true value of the characteristic measured will never be exactly known. This reasoning requires focus of attention not on the *observed values* but on *what these values represent* -- the larger family of all possible values of the characteristic being measured. The objective is to infer from the sample something about the population. Experience has taught that prediction (an inference) cannot be made with certainty. There is always a chance of being wrong. Errors of this type are called the prediction errors.

5.   FREQUENCY DISTRIBUTION

If all measurements referred to above are divided into small groups or cells having a range equal to about one-tenth the total range (from maximum to minimum) of all values, there will be about ten cells. Then if a count is taken of the number of values falling within the range of a particular cell, the ratio of this number to the total number of measurements available is the relative frequency of occurrence of measurements (events) in that cell. If the total number of measurements available is very large (1,000 or more) and *all values* falling within the cell are counted, a very good estimate of the true frequency of occurrence of values in that cell for that particular population will result. Doing this for all the cells would give values that could be plotted on a bar graph as follows: Arrange the cells along the abscissa in ascending order according to the magnitude of the midpoints of their range; erect bars over these midpoints with height proportional to the relative frequency in each cell and widths equal to the cell width. This bar graph is known as a histogram.

6.   NORMAL DISTRIBUTION

As the total number of values used is increased and the cell width (range) decreased, the step-wise form of the bar graph fades into a smooth curve that is called a frequency distribution. In practice, this is actually how a frequency distribution is formed. It means what the name implies. It is a distribution of (relative) frequencies.

Experience has shown that the families of values generated by repeated measurements of the same characteristic under controlled conditions have definite forms. The most common of these forms and the most useful is called the *normal* frequency distribution. This is the smooth curve described above. It is bellshaped. The family of values forming this distribution is called the normal population.

As the cell width in the bar graph decreases and approaches zero, the height of the bar represents the relative frequency for a single value on the abscissa. Thus there is a relative frequency for any value in the population of measurements. The sum of all the frequencies equals the frequency of all the values in the population which is assigned the numerical value of one. The equation for this function is known, but it is of no direct importance for the purpose of this discussion. It can be found in any standard text on statistics.

PROBABILITY

From a practical point of view relative frequencies (proportions) are estimates of probabilities. By definition, if it is certain that an event will occur, it is said that the probability of occurrence is equal to unity. If it is certain that an event will *not* occur, it is said that the probability of occurrence is equal to zero.

In the above example, if the cell width was equal to the range of the population (from the maximum to the minimum value in the population) it would be certain that the next value taken would fall within this "cell." As a result of taking repeated measurements, *all* of the values would fall within this "cell." The number of values falling within this "cell" divided by the total number of values will equal unity. That is, the probability of a value's falling within the "cell" (the event) is equal to one.

If, on the other hand, a new cell is taken having a maximum limit less than the minimum of the above population, it is certain that the next value taken from the above population will *not* fall within the new cell. If repeated measurements are taken from the above population, *none* of the values will fall within the new cell. The number of values falling within the cell divided by the total number of values will equal zero. That is, the probability of a value's falling within the cell (the event) is equal to zero.

The area under the normal frequency distribution is used to measure probabilities. As shown above, the magnitude of the ordinate associated with any value on the abscissa is a measure of the relative frequency of occurrence (or probability of occurrence) of that value. The summation of all the ordinates below any particular value on the abscissa is, of course, equal to the area under the curve below that ordinate. This area is then a measure of the probability of occurrence of all values in the population below the given value.

8. # PARAMETERS

Just two parameters or characteristics of the normal frequency distribution are required to define this curve completely. The first parameter is the central value around which most of the values belonging to a particular population will naturally cluster. This parameter is called the true or population *mean* and is measured by the arithmetic average of *all* the values in the population. The other parameter required is the dispersion of values around the central value. This parameter is measured by the root mean square of the deviations from the true mean and is called the true or population *standard deviation*.

9.   PROPERTIES OF NORMAL CURVE

Graphically, the mean is the ordinate that passes through the center of gravity of the area under the curve, since this curve is symmetrical. The mean is equal to the mode (the most frequently occurring value) and the median (the middle value).

Also, graphically, the standard deviation is equal to the horizontal distance between the ordinate of the mean and the inflection point on the curve on either side of the mean ordinate.
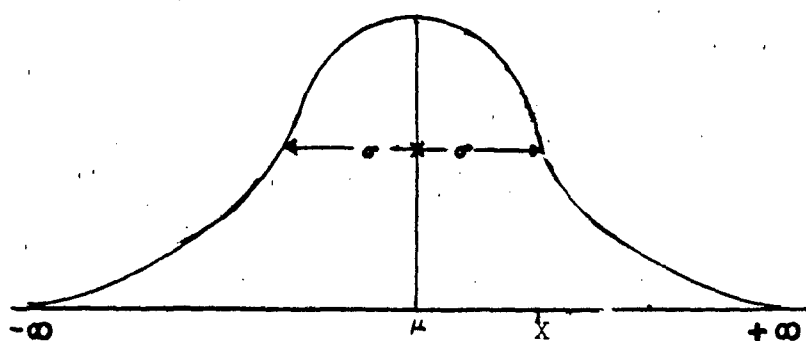
The Normal Deviate:
$Z = (x - \mu)/\sigma$
This is the linear measure of distance along the base of the curve in standard deviation units.

Where: μ =   The true population mean.

         σ =   The true population standard deviation.

         X =   Any observed value.

The true mean plus or minus one standard deviation includes 68.27 percent of the total area under the curve. The mean plus or minus two standard deviations will include 95.45 percent of the total area under the curve. These values are used to make probability statements. They mean either or both of the following:

A. In generating a normal family of values, 68 percent of the total number of values will lie within plus or minus one standard deviation of the mean. This is especially true if the total number is very large — i.e., 1,000 or more.

B. Randomly chosen values from the normal population have a 68 percent chance or a probability of 0.68 of falling within plus or minus one standard deviation of the mean.

This distribution is unique in nature. It is the curve of regression for the distribution of all small sample averages.

RANDOMIZATION

The meaning of the word *random* as used in modern statistics can be better described than defined. The phrase "randomly chosen values" describes a selection procedure of a very special kind. This procedure is free of biases of all sorts. It is the only procedure which will permit the free play of chance variations, which are the theoretical basis for all modern statistical techniques.

Random selection or random sampling can be accomplished by physically mixing the items before sampling, or by numbering all of the items and then using a table of random numbers to determine which items to select and in what order to select them. Random selection is the process used in lotteries, all numbered tickets being deposited in a revolving drum and a single drawing made by a blindfolded person. It is assumed that such a procedure is completely unbiased, that chance alone is at play, and that each ticket in the drum has an equal chance of being selected. The process of random selection then, not only permits the laws of chance to determine *which* item is to be chosen, but also the *order* in which successive items are chosen. This procedure relieves the experimenter completely of *any* responsibility concerning "which item" and "which order." In the lottery, the operator wants to be "fair." In an experiment, the experimenter wants to be unbiased.

11. SAMPLING

In a lottery the relative frequency of occurrence of any particular number is equal to the relative frequency of occurrence of any other number in the drum (population). Each number has an equal chance of being selected if the selection procedure is truly random and unbiased. However, the relative frequency of occurrence of the values in a normal population is not equal. Theoretically, all are different. A little reflection will show, however, that random selection will be "fair" and unbiased here, also. If in a bowl, 900 white beads and 100 red beads are mixed well (i.e., randomized), and a handful of beads selected by a blindfolded person, the ratio of red to white beads in that or any other handful will be close to 1 to 9. The average ratio of a large number of trials (handfuls) will be 1 to 9 -- the relative frequencies of the two colored beads in the bowl, the population. This same relation between sample and population holds true in selecting (sampling) values from a normal distribution if sampling is done in a random fashion. That is, every value (or item) in the population has a chance (probability) of being selected equal to the frequency with which it actually exists in the population. Only samples that can reflect these actual relative frequencies in the population can be considered as representing the population in an unbiased manner. Samples must correctly represent the population from which they are taken if valid inferences are to be made about the population, from the sample. Of course, successive samples drawn from the same population will not be identical, but if randomly selected, the difference between them will be due to chance errors only. Under these circumstances modern statistical techniques will identify them as having come from the same population, which, in fact, they did.

12. ESTIMATES

In practice, to make a measurement (or observation) is to estimate the true population *mean*. The more observations made and averaged together, the better the estimate. This estimate

is called a "point estimate" to distinguish it from an "interval estimate." However, it is assumed that the true mean is never known exactly unless an infinite number of observations is made.

If the root mean square of the deviations of the individual observations is calculated from the average of all observations, the true population *standard deviation* can be estimated. As with the mean, however, it is assumed that this true parameter is never known exactly unless an infinite number of observations is made.

13.   PREDICTIONS

The two predictions made most often in modern statistics are the following:

A. The magnitude of the true parameters. These predictions are based on interval estimates which are called confidence intervals.

B. Whether two or more values belong to the same population. These predictions are called tests of significance.

The prediction problem in modern statistics is to estimate first the population mean and standard deviation and then predict what these two population parameters might be or, given two or more estimates, to predict whether they came from the same population. If there are thousands of observations in each of the samples, the sample means and standard deviations are, for all practical purposes, equal to the population parameters and prediction becomes unnecessary. In practice, however, such large samples are generally not available. They are too costly to obtain. The problem, then, is to predict from small samples what the parameters might be or whether the samples came from the same population.

Intuitively, it is known that predictions cannot be made with certainty — there is *always* a possibility of being wrong. As a result, to be right as often as possible, reliance is placed on planning. In modern statistics, this possibility is maximized, and chances of being right are actually controlled.

To place this on a mathematical basis the assumption is made that the data have a normal frequency distribution. The normal distribution is then used to calculate the probability of being right in making predictions. This is called the *confidence level* of predictions. The techniques of modern statistics have been developed to make predictions. The assumption that the distribution is normal for variable (measured) data is a reasonable one. Experience has shown that the numerical values of measurable characteristics of products manufactured under controlled conditions are normally distributed (Ref. 11). In addition, the central limit theorem states that the distribution of averages of variable data is normal. So, when comparing averages or calculating confidence intervals of measured data, the assumption of normality is quite valid. This is true of attribute (counted) data only where they are transformed to variable data by some such process as the arc-sine transformation.

1.   PREDESIGN PHASE

A. Common Sense

Experimental plans and experimental results that violate common sense are discarded, not the common sense.

B. Past Experience

Use all available knowledge and information from past experience.

C. Choice of Variables

Make a comprehensive list of all the variables (factors or environmental treatments) whose effects on the components' functioning characteristics are of interest or must be determined. This should include:

a. Factors of direct interest.

b. Factors which may help show how the main factors work.

c. Factors required to determine the effect of experimental technique. In addition to choosing the variables to be used, their order of use must also be established. The order chosen should be the one most likely to be experienced in use or the one considered most severe. The order selected must be held constant throughout the experiment.

D. Choice of Factor Levels

a. Number of levels

The number of levels used in the designs described in this manual has been limited to two. These designs are the simplest and the most versatile for conducting multi-factor experiments.

b. Position of levels

In using only two levels those used are usually the extremes, such as the presence and absence of an environmental treatment or extremely low and high temperatures. The choice of levels used must be arrived at through the use of good judgement, common sense, and detailed knowledge of the purpose and probable outcome of the investigation. Factorial experiments are most efficient in their ability to detect differences among environmental effects when the levels of severity used are such that approximately 50 percent of the test specimens fail.

E. Scope

Consider the entire scope of the problem. Without regard to cost, time, or effort consider what it is that must be known eventually. If this turns out to be a very large experiment, the cost of which is prohibitive, divide the whole problem into rational parts. This makes possible a systematically planned approach. It also makes it possible to relate your test plan to cost and the amount of information required.

F. Possible Outcomes

Consider all possible outcomes and their physical interpretation. Results that have no physical interpretation have no practical value.

G. Choice of Criteria

Choose carefully the criteria on which conclusions will be based. To insist that a component have a reliability of 0.999 with respect to temperature shock is of little value when it has a reliability of only 0.80 with respect to transportation vibration.

H. Formulation of Hypothesis

Develop the right hypothesis by asking the right questions the experimental results are expected to answer. To show conclusively that component A has a much higher reliability than component B has solved nothing if component A cannot be mass produced.

I. Type of Measurement

The type of measurement to be used should be considered for the sake of efficiency. Variable type data can vary from minus infinity to plus infinity and furnish the maximum information per observation. Attribute data are "success" "failure" type data and furnish the least information per observation. From this it is clear that variable type data should be used wherever possible in factorial experiments; care should be exercised, however, in using variable data to determine reliability (see below).

J. Choice of Experimental Units

a. Definition of Experimental Unit

An experimental unit (test specimen) is the smallest sub-division of the experimental material that can receive different treatments.

b. Size of experimental unit

Sufficient homogeneous or uniform material should be available to conduct a complete set of treatment combinations (required by the experimental design) during a single period of time (such as a day) by a single instrument condition (such as calibration) and by a single operator or group of operators. Material produced during a particular period of time by a single process and by a single manufacturer can be considered homogeneous.

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c. Representative nature of experimental units

The experimental units used should not differ in any important respect from the best known (parts) design to which the conclusions are to apply. If design changes are made on the basis of experimental results, the items used to obtain t' . results are, of course, not representative of the modified design.

d. Independence of Experimental Unit

Experimental units should respond independently of one another. Obtaining a failure on one should not affect any of the others. Using a separate item for each treatment combination will usually assure independence.

K. Choice of Treatments

Treatments are chosen to give as direct an indication as possible of the functioning characteristics of the components and to include as many as possible of the environmental conditions expected in use. This is an engineering decision that must be based on good judgement and intimate knowledge of the purpose of the experiment.

L. Sequential Approach

The first experiment may have to be considered exploratory in nature. One or more ideas may be generated during the first experiment concerning parts design modifications or questions may be raised from the results of the first experiment concerning the exact effect of the environmental treatments. In either case additional experimentation would be required to:

a. Confirm the validity of the modified parts design.

b. Clarify the effects of the environments which produced the questionable results.

c. Include other treatments.

Committing oneself to a large experiment at the beginning of a new investigation may not be feasible. Small exploratory experiments may indicate a much more promising approach in a short time and with little cost. In this procedure the results of the first experiment are obtained and analyzed before the next experiment is designed.

2.   DESIGN PHASE

A. Choice of design

The factorial design and its modifications described in this manual meet the requirements of environmental testing experiments better than any other known design. The advantages of the recommended factorial designs for environmental testing are as follows:

a. Simple to use and analyze.

b. No control groups are required.

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c. The two levels of each treatment can be the presence and absence of the treatment, if desired. Alternatively, any two levels of the treatments can be used.

d. Each treatment effect can be determined independently of all the others. Unambiguous conclusions can be drawn about each treatment's effects.

e. Complex experiments involving a large number of treatments can be easily handled.

f. These are the only experimental designs with which the relationships among treatments can be measured. These designs can determine whether the effect of one treatment depends upon any of the others. These relationships are called interactions.

g. The probability of being right or wrong can be controlled.

h. When the number of treatments used becomes large (three or more), only a fraction (1/2, 1/4, 1/8, etc.) of the total number of combinations of treatments and levels need be used. These designs are called fractional factorials and optimum multifactorials.

i. A type of statistical analysis can be used that distinguishes between variations due to chance and variations having assignable causes.

j. More information can be obtained from a given number of test specimens than any other known procedure.

k. The effective sample size is increased by making it possible to use each observation (or measurement) for more than one purpose. In fact, each treatment effect is determined as though the entire experiment is conducted to determine that particular treatment effect alone. As a result, the precision with which each treatment effect is determined can be based on the total number of test specimens used in the experiment.

B. Sample Size

In any experimental situation a reasonable balance must be established between using too few test specimens thus obtaining poor precision, and wasting time and material in attaining unnecessarily high precision by using too many test specimens. When there is a preassigned number of test specimens available, the question is whether it is worthwhile to do the experiment at all. If the number of test specimens available is flexible and adequate, the number required for a given precision or reliability can be calculated in advance. The minimum number of test specimens required in the optimized designs is only one more than the total number of treatments used. The more versatile factorial designs require at least 16 items for five through eight treatments and at least 32 items for nine through thirteen treatments. With twice these numbers of items, the latter designs can also measure interactions.

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C. Orthogonality

The property of these designs, known as orthogonality, must be preserved in order to simplify the analysis and interpretation of the results. This can be done by keeping the number of observations per treatment combination equal and constant throughout the entire design. Orthogonality assures that all the environmental effects and their interrelationships can be independently estimated without entanglement.

D. Confounding

Confounding is the converse of orthogonality. It means confusing, entangling, or equating two or more factors or treatments so that their separate effects cannot be determined. For example, little can be concluded about the separate effects of the environmental treatments if all of the treatments are applied to each item. If a failure is obtained after an item has received two or more treatments, the cause of the failure is ambiguous; it could be the result of any of the following:

    a. The last treatment.

    b. The last two treatments.

    c. All of the treatments.

    d. Any of the other possible combinations.

The exact cause cannot be determined because the treatments are confounded. This type of confounding should be avoided.

E. Interactions

Interaction is said to be present when certain particular treatment combinations produce unusual results. This is the non-additive or unpredictable portion of the experiment; as such, interaction effects are considered discoveries by the U.S. Patent Office and as such are the only patentable portion of the experiment. When appreciable interaction effects are present, care must be taken in quoting main (average) effects. Any statement about the average effect of a treatment must specify the level of the interacting treatment associated with that average.

However, determination of interaction effects may be the most important information obtained from an experiment. It can explain what otherwise appear to be contradictions. This is the extra information furnished by factorials that cannot be obtained from other designs. Plans should be made to use factorials that can measure interaction effects if there is a possibility that they exist. Higher order interactions can be used as estimates of the error term when multiple replication is not used.

F. Replication

By replication is meant repetition. One complete replication consists of a single observation for each of the treatment combinations in the design. If the observations are performed in sets, so that a complete replication is done in a continuous period of time (such as a day), with a single measuring system (or instrument), by a single operator, the difference among replications can be used to determine whether the external experimental conditions have remained under control. Multiple replications are also used for the following purposes:

a. Increase the precision with which treatment effects are determined.

b. Furnish an independent measure of the error term.

c. As a basis for calculating the failure rate observed for each treatment combination in preparation for transforming attribute data to a continuous scale in analysis of variance procedures.

G. Blocking

In general, blocking means dividing the entire design into orthogonal sub-groups. This reduces the number of observations that need be taken in one continuous period of time and reduces the amount of homogenous material required in one batch. Differences among blocks due to uncontrolled changes with time and due to changes in material can be mathematically subtracted out of the system. That is, the object of blocking is to make it possible to conduct the experiment in reasonably small portions. Plans should be made to block any large experiment or any experiment expected to extend over a long period of time. Taking observations in complete replication sets is one form of blocking.

H. Randomization

Randomization can be accomplished by means of a table of random numbers or by drawing well shuffled numbered cards from a hat. The important characteristic of randomization is that it be an objective impersonal procedure. Proper randomization is determined by examining the procedure producing it, not by examining the results. To randomize does not mean to arrange in an order that looks haphazard. The object of randomization is to permit the laws of chance (probability) to have free play. Proper randomization is the most important requirement for a good experiment because it:

a. Prevents biased results of all kinds due to such things as, human prejudice, weather cycles, trends in time, heterogeneity of experimental material, etc.

b. Removes systematic error.

c. Relieves the experimenter of the responsibility of choosing which item to test or which test to conduct. Each item or test is equally likely to be chosen. In this sense the experiment is "fair" and unbiased.

d. Assures the validity of statistical techniques, such as the analysis of variance and associated tests of significance which depend for their validity upon the laws of probability.

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However, the use of randomization can be abused. Randomization should not be used to conceal large variations. This drastically reduces the sensitivity of the experiment to detect small differences. All variables known to have, or suspected of having, significant effects on the outcome of an experiment must be either controlled or designed into the experiment. The use of randomization should be considered as an expression of ignorance and used only to remove the effects of small variations after every other source of variation has been included in the design, or controlled. Only the use of good engineering judgment and a knowledge of the system can determine how, when, and where to use randomization.

3. ANALYSIS PHASE

A. Statistical Significance

The word significance has a special technical meaning in statistics. Its meaning must be understood in order statistically to analyze and interpret experimental results. One of the most important contributions of statistics is that it has established a means of distinguishing between chance variations and assignable causes. When the observed differences are due to chance variations, these differences are said to be non-significant. This means that the observed results originated from the same source (population). When the observed differences have assignable causes they are said to be significantly different. This means that the observed results have originated from different sources (populations). In a well planned experiment these sources can be identified. In the case of a non-significant difference, changing the treatment from its lower level to its higher level has not caused a detectable difference. In the case of a significant difference, changing the treatment from its lower level to its higher level has caused a detectable difference.

B. Interpretation

In a good experiment each treatment effect should have a unique interpretation. If two or more interpretations are possible, additional work is required to clarify the ambiguities. One of the most important requirements of a good experimental design is that the conclusions be unambiguous. Fortunately the factorial designs are very helpful in avoiding ambiguity. To conclude that an effect is not significant is not the same as saying that the effect does not exist. We can only say that there is insufficient data to detect the effect. However, if the conclusions are that the effects are significant (from the *test* of significance), we can be assured that the effect is real to the extent of the confidence level associated with the test of significance. Further advantages of factorial designs are as follows:

a. The range of validity of the conclusions concerning the average (main) effects is extended by the inclusion of more than one variable in the experiment.

b. Physical interpretation of interactions explain and clarify underlying mechanisms and relationships.

C. Qualitative Data (Success or Failure)

When only one observation is taken for each treatment combination, analysis of the results from the factorial designs described in this manual is made very simple by using the tables

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of minimum contrasts in Appendix 3A. These tables are based on the binomial distribution. The test of significance that uses the values in these tables is known as Fisher's Exact Method for 2 x 2 Contingency Tables. This test is valid even for small sample sizes and will determine not only the main effects but also the two-factor interaction effects when the proper designs are used (see example described below). When multiple (but equal number of) observations are taken for each treatment combination, the Fisher method can still be used. However, an alternate method which is slightly more efficient, but which requires more calculating can also be used. This method transforms the qualitative data to a continuous scale through the use of the arc sine of the proportion or percentage of failures found for each treatment combination. The transformed data can be analyzed by the usual analysis of variance techniques. The tests of significance and their interpretations are both made using the transformed data. If the arc sine transformation is considered desirable, it is suggested that a statistician be consulted to conduct the analysis of variance.

D. Quantitative Data

For quantitative data (such as g - values, voltages, or time) the usual analysis of variance can be conducted on the observed data provided the variances are homogeneous throughout the design. Since this procedure is somewhat involved, lengthy to describe, and is adequately covered in the literature (see ref. 18 and 19), an attempt will not be made to include the analysis of variance techniques in this manual. It is suggested that a statistician be consulted for this analysis.

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IV

PLANNING TEST PROGRAMS[a]

1.
STATEMENT OF THE PROBLEM

A. Identify the new and important problem area.

B. Outline the specific problem within current limitations.

C. Define exact scope of the test program.

D. Determine relationship of the particular problem to the whole research or development program.

2.
BACKGROUND INFORMATION

A. Investigate all available sources of information.

B. Tabulate data pertinent to planning new program.

3.
METHODS DEVELOPMENT

A. Hold a conference of all parties concerned.

a. State the propositions to be proved.

b. Agree on magnitude of differences considered worthwhile.

c. Outline the possible alternative outcomes.

d. Choose the factors to be studied.

e. Determine the practical range of these factors and the specific levels at which tests will be made.

f. Choose the end measurements which are to be made.

g. Consider the effect of sampling variability and of precision of test methods.

h. Consider possible inter-relationships (or "interactions") of the factors.

i. Determine limitations of time, cost, materials, manpower, instrumentation and other facilities and of extraneous conditions, such as weather.

j. Consider human relations angles of the program.

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[a]This outline was received in a private communication from Mr. Charles Bicking, Office, Chief of Ordnance.

4. DESIGN OF EXPERIMENT

A. Design the program in preliminary form.

    a. Prepare a systematic and inclusive schedule.

    b. Provide for step-wise performance or adaptation of schedule if necessary.

    c. Eliminate effect of variables not under study by controlling, balancing, or randomizing them.

    d. Minimize the number of experimental runs.

    e. Choose the method of statistical analysis.

    f. Arrange for orderly accumulation of data.

B. Review the design with all concerned.

    a. Adjust the program in line with comments.

    b. Spell out the steps to be followed in unmistakable terms.

5. DATA COLLECTION

A. Develop methods, materials, and equipment.

B. Apply the methods or techniques.

C. Attend to and check details; modify methods if necessary.

D. Record any modifications of program design.

E. Take precautions in collection of data.

F. Record progress of the program.

6. ANALYSIS OF DATA

A. Reduce recorded data, if necessary, to numerical form.

B. Apply proper mathematical statistical techniques.

7. INTERPRETATION OF RESULTS

A. Consider all the observed data.

B. Confine conclusions to strict deductions from the evidence at hand.

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C. Test questions suggested by the data by independent experiments.

D. Arrive at conclusions as to the technical meaning of results as well as their statistical significance.

E. Point out implications of the findings for application and for further work.

F. Account for any limitations imposed by the methods used.

G. State results in terms of verifiable probabilities.

V. COMPONENT RELIABILITY

1. INTRODUCTION

Reliability is the probability of the successful performance of a specified characteristic:

A. Under a specified condition or set of conditions,

B. For a specified length of time,

C. After a specified period of storage.

The "length of time" requirement can usually be included as part of the specified conditions.

The storage requirement has to do with age or storage life. This requirement involves the use of life-testing techniques. To be useful these techniques must be able to predict storage life from short-term (a few days or weeks) accelerated laboratory tests. In order for these predictions to be valid, the laboratory test results must be correlated with storage life results by actual long-term storage tests. At present this kind of information is not available.

Life-testing techniques can also be used to determine the reliability of an item with respect to environments whose level of severity can only be increased by increasing the length of time of exposure. To do this, however, requires the establishment of a minimum length of exposure time for successful functioning. The difficulty here is that component reliabilities established by tests of increased severity, in which time is the variable, are not comparable with component reliabilities established by tests of increased severity in which the level of the environment is the variable.

These two kinds of component reliability cannot both be used in the same system to calculate the reliability of that system and have the result meaningful.

The Ideal Test Condition

The ideal condition for determining reliability is that condition found in tensile or compression testing. That is, the following conditions exist which make possible the most efficient determination of the ultimate strength:

1. The observed results are in the form of variable-type data.

2. The severity of the applied stress can be easily increased until failure occurs.

3. The magnitude of the applied stress is continuously available so that the load at the point of failure can be directly observed.

4. The occurrence of failure can be detected by inspection.

5. The average of the observed results is an unbiased estimate of the ultimate strength.

With this combination of conditions and information the greatest precision and accuracy can be obtained with the smallest sample size. Aside from being convenient and easy to conduct, this method gives a direct measure of the ultimate strength and therefore the margin of safety from which reliability-in-use can be calculated.

The efficiency of variable-type data can be fully exploited here since each observed value is at the point of failure. This is the value of the stress that the 50% point on the cumulative frequency curve estimates in the indirect methods described in Section XI 3: Tests of Increased Severity. This average value at the point of failure in the ideal test and the 50% point in the indirect methods is important since it is the only unbiased measure of the ultimate strength, the margin of safety, and the reliability-in-use.

In all reliability testing the characteristics of the ideal testing condition should be kept in mind as a guide in more complicated situations where indirect methods must be used. In this way the disadvantages of testing without failure and collecting variable-type data at a single stress level can be seen in better perspective. For example, measuring the resistance of the circuits of several similar test specimens at a single voltage cannot measure reliability. This procedure gives only one point on the (I^2R)-strength curve. Where the point at which 50% of the items fail or what the margin of safety is cannot be determined using a single voltage value. Calculating the probability of obtaining resistance values outside given limits with information of this kind assumes that the margin of safety is equal to zero.

2.   COMPONENT TESTING

Component testing can be accomplished in either of two ways, controlled laboratory tests, or flight tests. Each of these has its advantages and disadvantages:

A. Advantages of controlled laboratory testing are:

a. Cost — This is the cheapest method both from the cost of test facilities and from the cost of test specimens for determining reliability with respect to separate environments during the development phase.

b. Information. — Complete information can be obtained since the test specimens are available for complete instrumentation and visual examination.

c. Controlled conditions — Each test specimen can be subjected to precisely the desired treatment.

d. Results — Unbiased estimates of reliability can be obtained by testing to failure in a predetermined manner so that the average reliability-in-use can be predicted from the test results.

e. Efficiency — Tests of increased severity can be used to demonstrate high reliability with small sample sizes.

f. System reliability prediction — Information can be furnished on a current basis during the development phase of an item which can be used as a guide during development and which can be used to predict the expected system reliability.

B. Disadvantages of controlled laboratory testing are:

a. Facility limitations — Environments must be applied in sequence instead of simultaneously as experienced in used.

b. __System reliability prediction__ — System reliabilities are predicted with incomplete information. The extent of component interaction and independence is not known. The degree to which the human factor, during assembly, reduces reliability is also not known.

c. __Sample size__ — Larger sample sizes are required for laboratory testing of components under use - conditions than for testing systems in flight to demonstrate a given systems reliability.

C. Advantages of flight tests are:

a. __Environment__ — Test specimens are subjected to actual use conditions; all of the environments are applied simultaneously and at the correct level of intensity and duration.

b. __Verification__ — Flight testing is a means of verifying all of the predictions based on component values and other information.

D. Disadvantages of flight tests are:

a. __Observation__ — The tested specimens are not available for examination.

b. __Measuring system__ — Measurement by telemetry is not precise or reliable.

c. __Cost__ — The cost of flying a test vehicle is excessive.

d. __Storage characteristics__ — Storage characteristics cannot be determined by flight tests.

E. From the above description of the relative merits of laboratory testing and flight testing the following conclusions can be drawn:

a. Laboratory testing furnishes the most information.

b. Efforts to improve testing methods should be directed to improving laboratory methods.

3. CALCULATION

A. __Tests of increased severity__

$$R = 1 - P$$

Where:

R = Mean reliability over the range of in-use conditions

P = Probability of failure-in-use measured by the overlapping areas under the stress and strength curves (see page 112 for curves) and which can be found by entering a table of areas under the standard normal curve (Appendix 3G) with the following normal deviate:

$$Z = \frac{(X_1 \cdot X_2) \cdot (M_1 \cdot M_2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

A failure can occur only when:

$$X_1 > X_2$$

Therefore the normal deviate becomes:

$$Z \geq \frac{M_2 - M_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

$X_1 = $ any stress value

$X_2 = $ any strength value

$M_1 = $ True (but unknown) mean of the stress distribution

$M_2 = $ True (but unknown) mean of the strength (failure) distribution

$\sigma_1^2 = $ True (but unknown) variance of the stress distribution

$\sigma_2^2 = $ True (but unknown) variance of the strength (failure) distribution.

The above values can be estimated from sample results as follows:

$$\hat{R} = 1 - \hat{P}$$

Where:

$\hat{R} = $ An estimate of the true reliability (R)

$\hat{P} = $ An estimate of the true probability (P) of failure-in-use which can be found by entering a table of areas under the standard normal curve (appendix 3G) with the following calculated value:

$$T \geq \frac{|\bar{X}_2 - \bar{X}_1|}{\sqrt{s_{11}^2 + s_{12}^2}}$$

Where:

$T = $ The normal deviate listed in appendix 3G for each $\hat{P}$ value.

$\bar{X}_1 = $ Average in-use condition (in terms of the environmental stress level) established by experience or actual measurement of the handling, storage, or flight conditions.

$\overline{X_2}$ = Average stress at the observed point of failure, or the stress at the point 50-percent point on the failure rate curve established by a test of increased failure when the occurrence of a failure cannot be determined by inspection.

$|\overline{X_1} \cdot \overline{X_2}|$ = Absolute difference between the two averages without regard to the algebraic sign, which is a measure of the margin of safety.

s_1 = Standard deviation of the in-use conditions (in terms of the environmental stress level) established by actual measurement of handling, storage, and flight conditions.

s_2 = Standard deviation of the stress at the observed point of failure or standard deviatic of the failure rate curve (in terms of the environmental stress level) established by a Bruceton-type test of increased severity when the environmental stress levels at the point of failure cannot be observed directly.

B. Life tests. When time is the variable rather than the level of the environment and the length of time (t_i) is observed for each failure and the test terminated at the exact time (t_a) of the last failure:

$$\hat{R} = e^{-t/m}$$

Where:

$\hat{R}$ = Sample reliability under the *test* condition as the probability of no failures in time (t).

e = 2.7183

$$m = \frac{t_1 + t_2 + \cdots + t_{\hat{a}} + (n-a) t_a}{a} = \frac{h}{a} = \text{Time per failure}$$

t_i = Time to failure of individual components.

h = Time during which "a" failures occurred.

a = Number of components that failed in time (h).

n = Number of components tested.

t = Required failure-free time.

This formula is not applicable during infant mortality or wear-out periods.

C. Binomial-type data (binomial distribution). The following technique is applicable when these conditions pertain:

a. The lot or population represented by the sample is very large or infinite.

b. The sample size is less than 10 percent of the lot size.

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c. Each test specimen can fail in only one way.

$$\hat{P} = \frac{k}{n}$$

Where:

$\hat{R}$ = Sample reliability. The probability of success under the *test* condition.

k = Number of successes.

n = Number of test specimens or number of trials.

D. <u>Variable-type data.</u> By definition reliability is the probability that an item will perform successfully under a specified set of conditions which can include environments, or time, or both. If it does not perform *successfully,* the item fails. By definition there are only two possible outcomes; success or failure. There are no other alternatives in reliability testing.* Half of the test specimens used can perform successfully, but any particular specimen cannot "only half succeed' or "succeed half way." Just as when tossing coins, heads can occur on half of the coins, but on any one coin there cannot be a "half of a head."

By definition then, there are only two possible outcomes in reliability testing. Data of this type — called attribute data — have only discrete values — are obtained by a counting process.

Variable data, as the name implies, can vary on a continuous scale-from zero to infinity. This type of data is obtained by a measuring process.

Text books on the subject of statistics state that variable data are more efficient than attribute data because more information is obtained per observation. But this advantage of variable data does not pertain to reliability testing except in the direct method where the observed values estimate the ultimate strength. If variable data be used for reliability testing in other cases, they can only be for the purpose of measuring a characteristic of the test specimen to determine the number of successes or failures. In this application, the "text book" efficiency of variable data is lost — the results obtained in this manner can be used in the formulas given above for calculating reliability.

The probability of a measured value's exceeding a given limit obtained from the average and standard deviation of a dependent variable (such as, ohms resistance, percent elongation, timing accuracy or hardness) at ambient static conditions does not measure reliability. There can never be a reliability with respect to a dependent variable. Dependent variables are properties of an item or material the same as reliability is a property.

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*Reliability testing means the stressing of a test specimen by an environment or time, to measure the margin of safety.

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An item or material cannot be stressed by, or subjected to, its own properties. In failure testing an item's properties can be used to determine only the number of successes or failures when the item or material is being stressed by or subjected to an independent variable such as, E.M.F. in volts, tensile load in pounds, or vibration in g's. In cases of this kind the observed proportion of successes measures the reliability with respect to the independent variable. The average and standard deviation of dependent variables at ambient static conditions can measure only the quality of material, the quality of the manufacturing process, or the effect of handling or storage on the measured properites — not reliability. Although there can be reliability with respect to storage conditions, this reliability must be measured using time as one of the independent variables in distributions such as the Poisson. Reliabilities of th's kind are stated in terms of the probability of no failures in a given length of time, not as the probability of a value exceeding a given limit.

VI   SYSTEMS RELIABILITY

1.   <u>INTRODUCTION</u>

        The advantages and disadvantages of laboratory and flight tests described above for com-
ponents also pertain to testing complete systems; however, testing complete systems cannot be used
as a prediction procedure during R and D since it is testing after the fact. In addition testing com-
plete systems is expensive and difficult even in the laboratory. As a result, it is concluded that com-
ponent testing must be done during the development phase in order to obtain the required detailed
information when it is needed. In so doing, all of the shortcomings of the several methods of reli-
ability testing which constitute the state of the art, culminate in the estimate of system reliability.
Some of the errors are compensatory, as:

        A. <u>Errors that underestimate reliability.</u>

            a. Testing without failure.

            b. Estimating reliability under extreme use conditions only.

        B. <u>Errors that overestimate reliability.</u>

            a. Applying environments in sequence instead of simultaneously.

            b. Estimating reliability with respect to only one environment.

            c. Calculating system relabilities on the assumption that components function and
react to environments independently.

        To what extent these errors compensate one another is not known.

        The one big advantage of testing complete systems under use conditions after the R and D
phase, such as flight tests during stockpile testing, is the higher reliability that can be demonstrated
in series systems with a given sample size. For example, if 15 adaption kits are flight tested without
a failure, this demonstrates a system reliability of at least 85 percent at the 90 percent (one-sided)
confidence level. On the assumption that the adaption kit is made up of 5 major components in
series, it would be necessary that 70 of each of the 5 major components be tested without a failure
to demonstrate an equivalent system reliability calculated from the components. In addition the
difficulty of how to apply the environments to the components in the laboratory would be en-
countered.

2. <u>CALCULATION</u>

Obtaining point estimates of system reliabilities from component reliabilities requires the development of a probability equation based on the circuitry of the system and the laws of probability. Since this procedure is treated extensively in readily available literature, such as reference 15, it has not been inc.. ded here.

When each test specimen (such as a system) can fail in more than one way, the Poisson distribution can be used as follows (ref. 14):

$$\hat{R} = e^{-\overline{X}}$$

Where:

$\hat{R}$ = Sample reliability under the test condition as the probability of no failures.

e = 2.7183

$\overline{X}$ = a/n the average number of failures.

a = Number of failures.

n = Number of test specimens.

3. <u>SAFETY</u>

Safety can be defined as the probability of a catastrophic failure. A measure of this characteristic can be obtained from the techniques described herein for reliability, with slight modification. For the determination of safety, only catastrophic failures can be counted and used. In this case, of course, the objective is to calculate the probability of a failure-in-use — not its complement.

The relation between safety and reliability can best be seen from the following diagram:

<u>PROBABILITIES</u>

Safe   Unsafe   ≈ 1

GOOD + DUD + PREMATURE + LATE ≈ 1

Reliable   +   Unreliable   ≈ 1

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1. Introduction

   A. Definition

          A confidence interval is a range of values within which the true population parameter-
such as reliability-is expected to lie.. The confidence level associated with this interval is a prob-
ability statement expressing the proportion of the time the true value is expected to be within the
interval or, is the probability of being right in predicting that the true value will be within the
calculated interval.

   B. Best Estimate

          In order to calculate a valid confidence interval the "best point estimate" of the true
population parameter must first be obtained. Whether an estimator is the "best" depends on how
it is determined and how it is used. On the assumption that the estimator used is the correct one
for the intended purpose, it is considered an unbiased point estimate if the mean of all the possible
sample values equals the true population parameter. This also means that the estimator is accurate.
In addition to being unbiased the estimator used should also be efficient. That is, the unbiased
estimator chosen for use should have the minimum variance of all the possible unbiased estimators
that could be used. This means that the estimator should be precise. An estimator that is both
unbiased (accurate) and efficient (precise) is said to give the "best estimate" of the true population
parameter. It is this kind of estimator that is required to calculate valid confidence intervals or
confidence limits.

          In most engineering work the arithmetic average of variable data is the "best estimate"
of the true mean of the population represented by the data. That is, the arithmetic mean meets
the requirements of a "best estimate" since:

          (1) The mean of all the possible sample (arithmetic) averages equals the true mean and is
therefore unbiased.

          (2) The variance of the arithmetic mean is smaller than those of other possible estimators,
such as the median, mode, or mid-range.

          What has been said above for variable data is also true for attribute data. This means that
in both cases the "best estimate" of the true population mean is the observed or sample average.

          In reliability testing the "best estimate" is the *observed* proportion of successes or
failures. Anything else cannot qualify as a "best estimate." For example, the various attempts that

have been made to avoid the dilemma created by obtaining no failures in a test sample include the use of the lower limit of the 50% confidence level as the "best estimate." This value cannot qualify as a "best estimate" since:

(1) The lower limit of a confidence interval can *rarely* be an unbiased estimate of the "true value" the interval is expected to encompass; (2) *any* lower confidence limit which equals or exceeds 50% has a larger variance than the observed value.

In summary, then, the *observed* sample average (or proportion) is the only value around which a confidence interval should be placed.

2. CALCULATION FOR COMPONENTS

A. For tests of increased severity

Calculate the limits of the confidence interval for $\overline{X}_2$ as follows (ref. 3):

$$\frac{\overline{X}_2 + ts_2}{\sqrt{n_2}}$$

Where:

$\overline{X}_2$ = Average stress of the failure distribution, or the stress at the 50 percent point on the failure rate curve generated by a test of increased severity.

t = Coefficient by which the standard deviation is multiplied to control the confidence level.

s_2 = Standard deviation of the failure rate curve (generated by a test of increased severity) in terms of the stress.

n_2 = Sample size used to obtained $\overline{X}_2$.

These adjusted values of $\overline{X}_2$ are then substituted for $\overline{X}_2$ in the above formula for reliability from tests of increased severity and the reliability recalculated for both limits. These recalculated values can be taken as the upper and lower limits of the confidence interval for the average (point estimate) reliability-in-use.

B. Life tests

When the length of time (t_i) is observed for each failure and the test terminated at the exact time (t_a) of the last failure.

$$e^{-Ut/2am} \leqslant R \leqslant e^{-Lt/2am}$$

Where:

R = True Reliability

e = 2.7183

U = Upper percentage point of the chi-square distribution obtained from Appendix 3c for half alpha and 2a degress of freedom.

L = Lower percentage point of the chi-square distribution obtained from Appendix 3c for one minus half alpha and 2a degrees of freedom

t = Required failure-free time.

$$m = \frac{t_1 + t_2 + t_3 + \cdots + t_a + (n-a)t_a}{a}$$

n = Number of components tested.

C. Attribute-type data

a. Binomial distribution:

The following technique is applicable when each test specimen can fail in only one way and when *any* one of these conditions pertain (page 120 ref. 21):

(1) The lot or population represented by the sample is very large or infinite.

(2) The sampe size is less than 10% of the lot size.

(3) Sampling is done *with* replacement.

Lower Limit (page 373 ref. 23):

$$p_1 = \frac{a}{a + (n-a+1) F_1}$$

Where:

p_1 = Lower limit of the confidence interval for defects or failures. One minus this proportion is the upper limit of the confidence interval for successes.

a = Number of defects or failures.

n = Sample size or the total number of trials.

F_1 = Upper percentage point from a table of the F-distribution.

Enter the F-table in Appendix 3E with the following values:

$$V_1 = 2 (n-a+1)$$

$$V_2 = 2a$$

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Upper Limit:

$$P_2 = \frac{(a+1) F_2}{(n-a) + (a+1) F_2}$$

Where:

P_2 = Upper limit of the confidence interval for defects or failures. One minus this proportion is the lower limit of the confidence interval for successes.

a = Number of defects or failures.

n = Sample size or total number of trials.

F_2 = Upper percentage point from a table of the F-distribution.

Enter the F-table in Appendix 3E with the following values:

V_1 = 2 (a+1)

V_2 = 2 (n-a)

These limits can also be obtained directly from the tables in Appendix 3B.

    b. Hypergeometric distribution:

This distribution is applicable when each test specimen can fail in only one way and when *all* of the following conditions pertain (page 120 ref. 21):

(1) The lot size is small (finite) but can be considered the population and not a random sample of a much larger volume of material.

(2) Sampling is done *without* replacement.

(3) The sample size exceeds 10 percent of the lot size.

The usual formula for the hypergeometric distribution calculates the probability that a given *sample* will contain exactly "x" defectives. This calculation is based on the size of the lot (population) when the *lot fraction defective is known.* However, the converse of this is usually required. Thus, knowing the observed fraction defective in the sample, the upper confidence bound of the fraction defective of the lot is required. Tables based on the hypergeometric distribution have been prepared from which the desired information can be obtained directly (see Appendix 3H). In addition, the upper confidence bound of the fraction defective of a finite lot can be estimated by multiplying the upper confidence bound of the fraction defective of an infinite lot by the following factor:

$$\sqrt{\frac{N-n}{N-1}}$$
                     (see page 121 ref. 21)

Where:

N = The lot size.

n = The sample size required to calculate a given upper confidence bound of the fraction defective in an infinite lot, using the binomial distribution.

Care should be taken in the use of the hypergeometric distribution. The upper confidence limit of the fraction defective of a finite lot is less than that for an infinite population when each is predicted from equivalent or identical samples. This comparison is shown in the following table for samples containing no defectives and for the 90 percent one-sided confidence level:

Sample Size	Lot Size	Proportion Defective	
		Hypergeometric*	Binomial**
2	40	.675	.684
4		.400	.438
8		.225	.250
16		.100	.134
32		.025	.070
5	100	.36	.369
10		.19	.206
20		.09	.109
40		.04	.056
80		.01	.028
10	200	.20	.206
20		.10	.109
40		.05	.056
80		.02	.028
160		.005	.014

* Finite lot size taken as the population.
**Infinite population.

The hypergeometric distribution is useful in acceptance testing where decisions must be made about specific lots of finite size. However, it should not be used in the development stockpile phases of a missile life cycle. In the development phase, decisions must be made about lots of indefinite size. In the stockpile phase, decisions cannot be limited to the small quantity in storage; at this stage of the life cycle, there is interest, also, in what the small stored quantity represents. That is, small quantities are placed in the stockpile to further the state of the art, not to win a war. For this purpose decisions must be made about the larger indefinite quantities represented by the stockpile. Predictions in this case require the use of the binomial rather than the hypergeometric distribution.

3. CALCULATION FOR SYSTEMS

A. Poisson Distribution

When each test specimen (such as a system) can fail in more than one way (ref. 14):

$$e^{-U/n} \leq R \leq e^{-L/n}$$

Where:

R = True reliability under the test condition, as the probability of zero failures.

e = 2.7183

U = Upper confidence limit of "c" (the counted number of failures) obtained from Appendix 3D.

L = Lower confidence limit of "c" (the counted number of failures) obtained from Appendix 3D.

n = Number of test specimens (systems) used.

B. Other methods

When the system reliability is calculated from component reliabilities, the lower bound of the confidence interval can be obtained by either of two methods recently developed at Picatinny Arsenal: One based on the propagation of errors method to calculate the variance (ref. 16) and one based on the Monte Carlo method of sampling (ref. 17). Both of these procedures are lengthy and involved. An electronic computer may be needed to make the calculations required by either of these methods.

However, before calculating a confidence limit for a system reliability the following should be considered:

a. Confidence limits based on biased estimates are also biased. The confidence level associated with such limits is not valid. Reliability values obtained under conditions that produce less than 50% failures are biased estimates of the true or ultimate reliability.

b. The magnitude of the differences between the nominal values (point estimate) of high reliabilities and the lower confidence limits based on their variances is always very small and of little practical importance.

c. If the lower confidence limit of a system reliability is to be determined, the method given above for the binomial distribution for components can be used. In this case the number of failures (a) equals $n(1-\hat{R})$ where ($\hat{R}$) is the system sample reliability and (n) is the average sample size which equals the sum of the component sample sizes used to determine ($\hat{R}$) divided by the number of component types (or kinds) that comprise the system. This procedure is quick and easy to calculate and is sufficiently accurate for most purposes.

d. Because of the efficiency of testing entire systems as a unit, pointed out above (Section VIII Systems Reliability), every effort should be made to test in this manner. This procedure also avoids the difficult problem of calculating the lower confidence limit of a system reliability derived from component reliabilities. Since the system is the experimental unit (or test specimen) in this case, the confidence limits can be easily calculated by either of the following methods given above:

(1) The binomial distribution when the system can fail in only one way.

(2) The Poisson distribution when the system can fail in more than one way.

The one exception to the rule of testing systems as a unit is in the development phase where a prediction procedure is required.

VIII   SAMPLE SIZE

1.   INTRODUCTION

Because of economic considerations, the question of how many specimens to test (or how large a sample size to use) is always given a prominent part in planning any testing program. It is the question most often asked by engineers concerning testing programs. To answer this question from only the economic point of view is not enough. The cheapest testing program is none at all! Of course, if no testing is done there is no verification that the newly developed item is useable and no information concerning the condition of a stored item.

Before the question of sample size can be answered, the following related points must be taken into consideration:

A. The notion that reliability is related to the number of specimens tested must be discarded. Only the *precision* with which the reliability is determined is related to the sample size.

B. There is no one single sample size that is applicable to all reliability testing programs. Each program must be considered individually.

C. A valid sample size cannot be stated without first knowing the *purpose* of the testing program. It is very easy to get the right answer to the wrong problem.

The purpose of planning the sample size prior to data collection is to obtain essential information with minimum cost, effort, and material, essential information being defined as the minimum information required such that additional data will not change the conclusions. To accomplish this the following design of experiment techniques must be considered, since the question of sample size cannot be answered out of this context:

2.   DESIGN OF EXPERIMENT TECHNIQUES IN SAMPLE SIZE DETERMINATION

A. Purpose and Objectives:

The purpose of any testing program is to verify the hypothesis that objectives (including requirements) have been achieved or maintained. To do so in any valid quantitative way, the characteristics of the sampling and testing procedures must be adequate, and to do so with the minimum sample size, these procedures must be highly efficient. By efficient is meant maximum precision with minimum sample size.

B. Precision of Sampling Procedures

In all practical testing programs, especially those in which the testing is destructive, something less than all of the existing items should be tested and from this an inference made about the remaining (usually larger) portion of items. To have these inferences valid the sample must "represent" the remaining portion of the lot or population. If the lot is homogeneous, a representative

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sample can be obtained by random selection. That is, each individual item in the lot must have equal change of being selected. If the lot is not homogenous but stratified in some manner according to geographical location, weapon, or manufacturing process, then the sampling plan must be designed to cope with this characteristic of the lot. If the strata are only few in number, then an equal number of randomly selected specimens should be selected from each stratum. The number selected should be apportioned according to the size or importance of each stratum. If the number of strata is large, then specimens should be taken from only a part of the strata. If all strata are equivalent, then a sample of the total number of strata should be randomly selected before the specimens within them are randomly selected. If all of the strata are not equivalent, then the most important or largest strata should be used. In any case, the actual selection of strata or specimens must be done in a random manner either by physical mixing and selection, or by numbering and determining which numbers to select by means of a table of random numbers.

It is important that the sample be stratified correctly to parallel that of the lot. It is only in this way that the heterogeneity of the sample can be kept to a minimum. Any increase in the heterogeneity of the material due to sampling, results in an inflation of the overall variation as measured by the standard deviation of the testing method. As shown below, the magnitude of the standard deviation is of prime importance in calculating sample size.

C. Precision of Testing Method

The testing method is in reality a measuring system or device. It "measures" the characteristic of the item being used as a basis for evaluation and decision. As any measuring device, the testing method must be precise and accurate. By precise is meant that characteristic of the method that produces estimates (numerical results) from repeated trials that are close together when in fact there has been little or no variation in the system. Methods which produce estimates close together and do not reflect variations that actually occur in the system are called insensitive rather than precise methods. Obviously this type of method is to be avoided.

It is important to have available the most precise methods possible, since, as can be seen below from the formulas, the sample size varies directly as the square of the standard deviation. The only way precise methods can be made available is via a continuous program of methods development.

D. Decision Errors

Because all testing is done on a sample basis, decisions (inferences) must be made about the lot based on the information gained from the sample. This in reality is a form of prediction. We all know that predictions cannot be made with certainty. However, there are only two kinds of error that can be committed in drawing inferences about the lot:

    a. Type I error is rejecting good material.

    b. Type II error is accepting poor material. It is desirable to keep both of these errors small. Their magnitude can be controlled by the number of specimens (sample size) tested in any given situation, as shown below. In practice, the magnitude of these errors *chosen* is based on the consequences of being wrong. (For example, the consequences of rejecting good material (Type I error) can cost only dollars, but the consequence of accepting poor material (Type II error) can cost lives and lose wars.) Then, the sample size required to maintain both errors at the selected levels is calculated.

E. The Difference that Must be Detected

In a testing program of any kind a decision must be made about at least one of the following requirements before anything can be said about sample size:

    a. The maximum confidence interval that can be tolerated for the particular purpose intended.

    b. The minimum difference (between two values) necessary to be detected for the purpose intended.

These requirements can be established only through knowledge of the objectives and purposes of the system under consideration. Fortunately, this kind of information is usually well known to the engineer. Sample size varies inversely as the square of the difference to be detected. The sample size required to detect a difference of (d/2) is four times that required to detect a difference of (d).

F. Experimental Design

a. Multi-variable Experiments

If effect of more than one variable (such as effect of more than one environment) must be determined, experimental design is extremely important in keeping sample size to a minimum. (By experimental design is meant the pattern or combination of the variables used to collect data.) If these combinations are correctly chosen, efficiency of the experiment can be greatly enhanced. In fact, the efficiency is improved by a factor equal to the number of variables included in the design. For example, to obtain a given precision, a factorial design for three variables requires only one-third the number of test specimens required by the classical one-at-a-time procedure. Factorial design for seven variables requires only one-seventh the number of test specimens required by the classical one-at-a-time procedure. (A factorial design is an experimental one in which all possible combinations of the variable levels are included in the experiment.) At least one test specimen is required for each combination used. When this number becomes large, only a part of the total number of combinations need be used in designs called fractional factorial designs. These fractional designs have the same high efficiency as the full factorial designs. These designs should be used to screen all of the variables of interest to find the most important ones - such as the most severe environment.

b. Test of Increased Severity

Because of the exploratory nature of this test, an overall sample size cannot be precisely predetermined. The number of test specimens required to obtain the first failure depends upon the magnitude of the existing safety margin and the magnitude of the increments of stress used. After the first failure is obtained, the effective sample size is equal to only one-half the total number of test specimens used in the Bruceton up-and-down and the Two-Stimuli methods. However, these methods are highly efficient. That is, high reliabilities (if they exist) can be demonstrated with small sample sizes. For example, a reliability of .995 at the 90% (one-sided) confidence level can be demonstrated with 40 to 50 items with these methods. Higher reliabilities would require no larger sample size. In addition, these methods make it possible to calculate the reliability under the use condition.

Without these methods the above reliability would require at least 500 items tested without a failure. The reliability demonstrated in this manner would be under the test condition only. It would not be possible to calculate the reliability under the use condition. To demonstrate higher reliabilities (if they exist) would require larger sample sizes.

c. Life Test

When time is the variable instead of the environment, as in storage during Stockpile programs, the Poisson distribution is applicable. In this case, the sample size required to demonstrate a given reliability is directly proportional to the ratio of the required shelf life to the length of storage at the time of testing. If this length of storage is short compared to the expected shelf life, very large sample sizes with very few failures are required to demonstrate a reliability above 0.90.

G. Confidence Intervals

A confidence *interval* is defined as that interval around a sample value (such as the average) in which we expect the true (population) value estimated by the sample to lie. The confidence *level* is a probability statement expressing the proportion of the time the true value can be expected to be within the stated interval. The confidence level is the complement of the Type I error. That is, one minus alpha equals the confidence level. Where alpha is the probability of being wrong (in error), the confidence level is the probability of being right in our predictions. These probabilities can be measured by the area under a frequency distribution curve, such as the normal curve. As a consequence, there are two ways in which an area equal to alpha can be cut off.

    a. By cutting off an area equal to alpha all in one tail of the curve.

    b. By cutting off an area equal to one half of alpha in both tails of the curve.

Either way, the confidence level is the same for a given alpha value. To distinguish between these two ways the first is called a "one-sided" or "one-tail" level, and the second way is called a "two-sided" or "two-tail" level.

The one-and two-sided confidence levels have distinctly different uses:

    a. If there is interest in only one confidence limit, the one-sided level should be used.

    b. If there is interest in both confidence limits, the two-sided level should be used.

The decision concerning which type of confidence level and what magnitude of confidence level to use in any given situation must be made *prior* to obtaining the data. The type of level must be based on the need for one- or two-limit intervals, and the magnitude of level must be based on the consequences of being wrong. To make these decisions after seeing the data affects the value of the confidence level associated with a given confidence interval. Probability statements derived from a set of data are not applicable to that set of data. The fact that one-sided confidence levels for reliability are higher than two-sided levels is not a valid reason for choosing one-sided confidence levels.

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With these considerations in mind, the sample size required for a given confidence interval can be calculated as shown below. Conversely, for a given sample size, the magnitude of the associated confidence interval can be calculated. However, for variable type data the standard deviation must be known.

H. Testing Hypotheses

Tests of hypotheses are used to compare two or more values, such as reliability values. The purpose of tests of this kind is to determine whether observed differences are due to chance variations or whether they are due to assignable causes. This is important in decision making. To decide that the reliability value obtained during the second testing period is smaller than that obtained during the first testing period is very disconcerting if the value obtained in the third testing period is larger than the first reliability value obtained. This is especially disturbing if the decision has led to more testing or replacement of parts, but this is exactly what can happen if the observed differences are due to chance variations. Only through use of statistical tests of significance can this difficulty be avoided.

In hypotheses testing both the alpha (Type I) error and the beta (Type II) error should be controlled to prevent difficulties of the kind described above. The beta error is especially important in Ordnance work because of the consequences of being wrong. The only way that these errors can be controlled at predetermined values is to calculate the sample size required to do so in advance of data collection. Experiemce has shown that when these two kinds of errors are kept at 5% or below, the risk of making a wrong decision is sufficiently low for most purposes. To reduce these errors below 1% requires very large sample sizes.

I. Other Considerations

For lots made up of discrete items from which only attribute (success or failure) data can be obtained the following additional considerations should be made:

a. Lot Size vs Sample Size

If the lot is finite in size and less than ten times the size of the sample selected then this fact must be taken into account. The action taken in this regard depends upon the purpose of the testing and the scope of the conclusions drawn as described below.

b. Disposition of Selected Specimens

If the testing done destroys the specimens selected or if the specimens are not returned to the lot for any other reason, this fact must be taken into account. Again the action taken in this regard depends upon the purpose of the testing and the scope of the conclusions drawn as described below.

c. Purpose of Testing

A decision should be made prior to data collection concerning the purpose of the testing. If the purpose is to draw a conclusion about only those items in a small (finite) lot

and if both of the two above conditions pertain, then the sample size can be reduced slightly through use of the hypergeometric distribution. This distribution finds its most frequent use in acceptance testing where the purpose is to predict the expected fraction defective of a particular small lot of items. If such a lot is placed in stockpile, however, then the hypergeometric assumption is no longer applicable since the purpose of testing is now different. Small lots of material in the stockpile represent larger lots of indefinite size. The characteristics of the material are studied and recorded for their value in future applications. That is, the purpose of testing is to draw inferences about the larger volume of material represented by the small lot on hand. In this latter case, only the binomial assumption concerning lots of infinite size is applicable.

3. CALCULATION

A. Sample size required for a given confidence interval

a. Variable data

$$n = \frac{(ts)^2}{d^2}$$

Where:

n = Sample size

t = Standard deviate associated with the alpha error used to control the confidence level.

s = Sample standard deviation.

d = Magnitude of the confidence interval in the same units as the standard deviation.

b. Attribute Data

(1) Binomial Distribution

There is no easy, practical way accurately to calculate the sample size required for attribute data. The accurate methods are difficult to calculate and the simple, easy methods are not accurate. The most practical method is to refer to one of the existing tables for binomial confidence intervals to find the sample size required for a given interval. Tables useful for this purpose are:

One-Sided Limits:

Appendix 3B

Two-Sided Limits:

Appendix 3B

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(2) Hypergecmetric Distribution

As with the binomial distribution, there is no easy, direct way to calculate the sample size for the hypergeometric distribution. The most practical way to arrive at a sample size in this case is to refer to one of the existing tables for the hypergeometric confidence intervals. From these tables the sample size for a given interval and confidence level can be read directly. Tables useful for this purpose can be found in Appendix 3H.

Alternatively, the sample size required in a hypergeometric distribution can be estimated by multiplying the sample size for the binomial distribution by $N/(N+n)$.

Where:

N = Lot size

n = Sample size required in the binomial distribution.

B. The sample size required to detect a given difference between two sample values in testing a hypothesis:

a. Variable Data

$$n = 2 \left[\frac{(t_1 + t_2)}{d} \right]^2$$

Where:

n = Sample size

t_1 = Standard deviate associated with the alpha error

t_2 = Standard deviate associated with the beta error.

s = Sample standard deviation

d = Difference that must be detected.

b. Attribute Data

As mentioned above there is no easy practical way to calculate the sample size for attribute data. The sample size for hypothesis testing using attribute data can best be determined from the tables for minimum contrasts in Appendix 3A. These tables give values in the following format:

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Minimum Contrasts Required for Significance at the 95% Level

N	No. of A's in sample (1)/No. of A's in Sample (2)			
4	0/4	1/-		
5	0/4	1/5	2/-	
10	0/5	1/7	2/8	3/9, etc.
20	0/5	1/7	2/9	3/10, etc.

In this table N is the sample size. The values in the body of the table that appear to be proportions are written in a short-hand method which mean the following:

For a sample size of 5, the value in the first column of this row (0/4) means that if no failures are obtained in the first sample of 5, at least 4 failures must occur in the second sample of 5 before the observed difference can be declared significant at the 95% level of confidence. This, in turn, means that a sample size of 5 can only detect differences of 80% or greater. Larger sample sizes can detect smaller proportional differences. The use of these tables can, of course, be reversed to find the sample size required to detect a given difference.

C. Sample size required in storage programs where time is the variable:

$$N = \frac{a}{1-R}$$

Where:

N = Sample size

a = Number of failures in time (h)

h = Length of storage

R = Required reliability

1.   INTRODUCTION

It is assumed in these methods that the test item can fail in but one way. That is, the binomial distribution is applicable.

Plans should be made to conduct the laboratory experiments in two stages:

A. Survey the separate effects of the several environmental conditions of interest in one integrated factorial experiment to select the environments causing the highest failure rates.

B. Determine the ultimate reliability by means of a test of increased severity (testing to failure) using the treatment (environment) found most severe in the factorial experiment.

2.   FACTORIAL DESIGNS

A. Advantages

The two-to-the-n^{th} factorial designs or their optimized modifications are the most efficient experimental methods known for selecting the treatments causing the highest failure rates. This approach will reduce the magnitude and complexity of the experiments required to determine reliability. More important, all component reliabilities obtained in this manner will have a common basis of determination because the reliability of each component is defined in terms of the environment which has been experimentally found to cause the highest failure rate. This results in predicting the minimum reliability with respect to the separate environments for each component. If all these reliabilities are acceptable the reliabilities associated with all the other environments will also be acceptable. Only in this way can valid and realistic system reliabilities be derived from component reliabilities.

See Appendix 4 for some of the more useful two-to-the-n^{th} factorial designs in the form of worksheets. These designs are the most efficient known. Experiments based on these designs may be conducted without changing the treatment procedure except to arrange for the test specimens to receive the number and kind of treatments required by the particular design used. However, the best differentiation among treatments is obtained when the level of severity used will cause 50 percent of the test specimens to fail. This may cause some adjustment of the levels of the treatments used.

For the purpose of this application, the two levels of each treatment can be the *presence* and *absence* of the treatment. Alternatively, any two levels of the treatment can be used.

The number of test specimens required in the optimized designs is one more than the total number of treatments used (ref. 5). The more versatile fractional factorial designs (ref. 6) require at least 16 items for experiments containing from five through eight treatments, and at least 32 items for nine through 13 treatments. With twice these numbers of items, the latter type designs can also measure interactions, i.e., how the effect of any one environment depends upon the others. Interactions among treatments cannot be measured except by factorially designed experiments.

Factorial designs permit a type of statistical analysis that distinguishes between variations due to chance and variations having assignable causes, thereby producing more information from a given number of items than any other known procedure. These designs actually increase the effective sample size by making it possible to use each observation (or measurement) for more than one purpose.

In fact, each treatment effect is determined as though the entire experiment is conducted to determine that particular treatment effect alone. As a result, *each* treatment effect is determined with a precision equal to the total number of items used in the experiment. The three-treatment-design example described below demonstrates this point.

Further advantages in using factorial designs in environmental testing experiments follows:

a. No control groups are required.

b. Each treatment effect can be determined independently of all others. Thus, unambiguous conclusions can be drawn about each treatment effect.

c. Complex experiments involving a large number of treatments can be easily handled with factorial procedures.

d. This is the *only* experimental design in which the relationship among the treatments can be measured. The factorial design can determine whether the effect of one environmental treatment depends upon *any* of the others. These effects are called interactions.

e. The probability of being right or wrong can be controlled.

f. When the number of treatments used becomes large (three or more), only a fraction (1/2, 1/4, 1/8, etc.) of the total number of combinations in a factorial design need be used.

When multiple replications cannot be used and only attribute (go, no-go) data are available, these designs can still be used to take advantage of their efficiency. However, in cases of this kind the usual analysis of variance cannot be made. Instead, the usual summations are made to obtain and compare two binomial proportions (by the Fisher exact method) to determine the effect of each treatment. See example No. 1 below.

Results of factorial experiments are used as a guide to select which environment to use for determining reliability prior to conducting the test of increased severity. The factorial experiment surveys all of the environmental treatments of interest (with a minimum number of test specimens) to determine the difference, if any, among the environmental effects. A decision is then

made whether to redesign the item. If the item is considered acceptable at this time, reliability is determined using the environmental treatment or treatments found to be most severe. If no differences are found among the effects, reliability can be determined by using a combination of several of the treatments considered most important from an engineering point of view. If reliability is determined by using the most severe treatments, the reliability values obtained will be lower than those obtained with the other treatments. This is a necessary condition if the system's reliability derived from the component's reliabilities is to be useful.

B. Full factorial designs (ref. 18).

These designs require more specimens per treatment than do the fractional factorial designs, but they are the only class of designs that can measure *all* of the interaction effects. A full factorial can be formed by writing down all of the combinations of "n" treatments, each at two levels in a multi-entry table. For example, a full two-cubed factorial can be written as follows:

2^3 FACTORIAL DESIGN

	A_1			A_2	
	B_1	B_2		B_1	B_2
C_1	(1)	b		a	ab
C_2	c	bc		ac	abc

The lower case letters and the symbol (1) in the body of the table identify each of the eight ($2^3 = 8$) treatment combinations that constitute this design. These combinations are derived from their position in the table. For example, the symbol (1) is located by $A_1 B_1 C_1$ which means that all three treatments are at their lower level. The lower case letters (ac) are located by $A_2 B_1 C_2$ which means that treatments A and C are at their higher levels and that treatment B is at its lower level. In this code the lower case of the treatment letter appears in the combination only when the treatment is at its higher level. This results in the formation of all possible combinations of "n" things (treatments) taken 0, 1, 2, ... and n at a time. At least one test specimen or observation is required for each of the treatment combinations. Two or more observations at each treatment combination are required for an independent estimate of experimental error. An equal number of observations at each treatment combination is required to keep the design orthogonal.

C. Fractional factorials (ref. 6)

As the number of treatment variables increases, the number of treatment combinations, and therefore the number of test specimens required for a complete replication, increases very rapidly. At the same time the number of higher order interactions that can be measured also increases very rapidly. This results in two undesirable situations:

a. The number of test specimens required is too large.

b. The information in the higher order interactions (three-factor interactions and above) is of little practical use. Fractional factorial designs were developed to avoid these situations and thereby improve the efficiency of designs for multi-factor experiments.

When less than all of the possible combinations in a factorial design are used, the design is said to be a fractional factorial. For the two-to-the-n^{th} series there can be half, quarter, eighth, sixteenth, etc. portions of the full factorial used. These portions are called fractional replicates, where a full factorial is one replicate.

Fractional factorials cannot be used without losing or giving up some information that is available in the full factorial. However, it is planned in designing a fractional factorial to lose only the least important part of the information. Experience has shown that the higher order interactions in a full factorial are the least important. This fact is made use of by equating new treatments to the higher order interactions. To equate one such interaction to a new variable in a full 2^6 factorial, for example, creates a half replicate of a 2^7 factorial. Detailed procedure for designing fractional factorials can be found in reference 18. At least one observation for each treatment combination is required to keep these designs orthogonal.

D. Treatment procedure

The factorial designs described in Appendix 4 are those most frequently used in environmental experiments. They are described in the form of treatment procedure worksheets to facilitate their use. These worksheets show, in an easy-to-follow manner, how to treat each test specimen in the various fractional factorial designs represented. They can also be used to record and analyze the test results. A blank space in the item column means that the item does not receive the corresponding treatment. A plus mark in the item column means that the item receives the corresponding treatment. The combinations of blank spaces and plus marks in the worksheets correspond to the treatment combinations in the respective fractional factorial designs. The choice of these designs should be based on the following considerations:

a. The number of treatment effects that must be determined.

b. Whether interactions can be expected to be present.

c. The precision required.

d. The number of test specimens available or that can be made available.

These considerations should be made in the order named.

The blocks in which some of the designs are divided are for the primary purpose of breaking the experiment into homogeneous parts with respect to testing equipment used, operators conducting the experiment, or climatic conditions, such as season of the year, etc. If all such things can be considered constant, then these blocks can be identified with other conditions whose effect it is desired to evaluate, such as, firing conditions, functioning conditions, temperature conditions, or different lots of material. Identifying the blocks with different conditions or material does not affect the determination of the treatment effects. The important consideration is that conditions be held constant and materials be homogeneous within the block.

E. Analysis

An example of one type of analysis that can be used with factorial designs is given in Appendix 1. This is the simplest possible analysis. The type of analysis that can be made depends upon the class of design used, the kind (attribute or variable) and amount (number of replications) of data, and the way (at random or in blocks) data were collected. Some types of analysis, such as the analysis of variance, are quite complicated. As a result, the subject of the analysis of variance (ref. 18) is not included here. It is recommended that statistical analysis of this kind be conducted by statisticians.

3. TESTS OF INCREASED SEVERITY

A. Introduction

These methods need be used only when one of the following situations pertains:

a. The occurrence of a failure cannot be detected by visual inspection at the time of occurrence, as it is in a tensile test.

b. The magnitude of the stress at the time of failure is not observable, as it is in the tensile test.

The intended use of these methods is to determine the magnitude of the stress at the point of failure (where the stress equals the strength), when this value is not directly observable, as in the case of the effect of vibration on timing accuracy.

The level of severity can be increased in a variety of ways, such as the following:

a. Using more extreme levels of treatment (e.g., higher or lower temperatures, higher or lower G-values, or higher or lower voltages).

b. Applying two or more treatments simultaneously.

c. Increasing the length of time the treatment is applied, as in storage tests.

When variable (quantitative) data (such as resistance in ohms, elongation in percent or closing time in seconds) are obtained, it is necessary to compare each observed value with the required value in order to determine success or failure.

B. Bruceton up-and-down method (ref. 3)

Starting with the most severe condition expected in use, test one new, unused item. If the item does not fail, increase the level of severity (the stress) one increment* and again test one

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*This value can be estimated by dividing the difference between the maximum, and minimum in-use conditions by six. This is based on the assumption that the extreme in-use conditions are the 3-sigma limits.

new, unused item. Continue this process of increasing the stress one increment at a time and testing one new, unused item at each increment of stress until the first failure is obtained. Then reverse the process by decreasing the stress one increment at a time and testing one new, unused item at each increment of stress until a success is obtained. Repeat the process of increasing the stress to failure and decreasing the stress to success until at least 25 test specimens are used after the first failure. Calculate the level of severity at which 50% of the specimens fail, and the associated standard deviation by the method described in Chapter 19 of ref. 3, using the number of failures for these calculations.

With this information the "reliability-in-use" can be predicted. See the examples in Appendix 1 for details of the calculations.

When the form of the distribution curve is not known or is in doubt, Chebyshev's inequality can be used. This technique is valid for *any* distribution without an assumption concerning its form. The inequality states that the amount of area under *any* distribution curve which is farther away from the mean than k standard deviation units is less than $1/k^2$. The reliability calculated by this procedure will always be less than the true value.

When the form of the failure distribution curve is practically normal, as shown by its cumulative frequency approximating a straight line on linear probability paper, probability values can be found by entering a table of areas under a standard normal curve with calculated normal deviates, which equals the difference between any two levels of severity divided by the standard deviations.

C. Churchman two-stimuli method (ref. 10)

Test one new, unused item at the most severe condition in use. If the item does not fail, increase the level of severity (the stress) one increment* and, again, test one new, unused item. Continue this process of increasing the stress one increment at a time and testing one new, unused item at each increment of stress until the first failure is obtained. This procedure should cause the first failure within 5 to 10 trials, depending on the magnitude of the safety margin. Using the level of severity causing the first failure, test 10 to 20 items to determine the proportion of failures at this point. Record this proportion and the level of severity used. Then change the stress by an amount equal to about two or three increments. If the first proportion of failures exceeds 50 percent, decrease the stress, and if the first proportion is less than 50 percent, increase the stress to *find a second point on the curve.* Determine the proportion of failures at this point as before and record this proportion and the level of severity used.

The object is to find two levels of severity such that the proportion of failures differ by at least 20 percent, and yet have the proportions more than zero percent and less than 100 percent. From this information calculate the average and standard deviation of the failure rate by the method described in Reference No. 10. Alternatively, the average and standard deviations can be obtained graphically by plotting the proportion of failures against the corresponding stress level on linear probability paper. Draw a straight line through the two points. The average stress is that stress corresponding to 50 percent failures. The standard deviation is equal to the difference

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*One sixth of the difference between the expected maximum and minimum use conditions.

between the stress at the 16 percent point, and the stress at the 50 percent point. By using these values, the reliability-in-use can be calculated as described in Appendix 1.

D. Discussion of methods.

Which of these methods will be suitable for use in any particular situation depends upon the intended purpose of the experiment. The choice can be based upon the distinguishing characteristics. Both methods are equally efficient, as they both require the same sample size for a given precision.

The two-stimuli method should be used when either of the following physical conditions exists:

a. The test results are not immediately available after each trial. This would cause undue delay in conducting the Bruceton method which requires that all trail results be known before the condition for the next trial can be determined.

b. The physical changing of the test conditions is difficult. This would cause undue work in conducting the Bruceton method which requires changing the test condition after each trial.

E. Method characteristics

a. Bruceton method

(1) Advantage:

This method leads directly to the 50 percent point with the greatest efficiency.

(2) Disadvantages:

(a) The standard deviation should be known in advance.

(b) Tests must be conducted in sequence, as the results of each test must be known before the next is conducted.

(c) Test conditions must be changed after each trial.

b. Two-stimuli method.

(1) Advantages:

(a) A number of trials can be conducted concurrently.

(b) Only two points on the curve are required.

(c) This method can be extended so that more than two points are determined. If this is done the form of the distribution can be determined.

(2) Disadvantages:

    (a) The form of the strength distribution curve cannot be determined with only two points.

    (b) The assumption of normality is required when only two points are used.

X APPENDICES

APPENDIX I

EXAMPLES

A. Confidence Intervals

a. Life tests. A sample of 20 components (n) which are required to operate for 240 hours (t), were subjected to a specified use condition for a period of 120 hrs. when the first component failed. The failure rate was assumed to be relatively constant and so the test was discontinued at this point in time (120 hrs.).

The sample point estimate for reliability can be caluclated as follows:

$$\hat{R} = e^{-t/m}$$

When: $\quad$ e = 2.7183

$$m = \frac{1 \times 120 + (20-1)120}{1} = \frac{2400}{1}$$

$$t = 240 \text{ hours.}$$

$$\hat{R} = (2.7183)^{-240/2400} = .90$$

This is the point estimate of the probability of no failures in 240 hours. The 90 percent two sided confidence interval for R can be calculated as follows:

$$e^{-Ut/2am} < R > e^{-Lt/2am}$$

When:

e $\quad$ = $\quad$ 2.7183

U $\quad$ = $\quad$ 5.99 (from Appendix 3C for half alpha and 2a degrees of freedom)

L $\quad$ = $\quad$ 0.103 (from Appendix 3C for one minus half alpha and 2a degrees of freedom)

a $\quad$ = $\quad$ 1 (the number of failures)

m $\quad$ = $\quad$ 2400

t $\quad$ = $\quad$ 240

alpha $\quad$ = $\quad$ (1-0.9) = 0.1

$$(2.7183)^{-(5.99)(240)/2 \times 1 (2400)} \leqslant R \geqslant (2.7183)^{-(0.103)(240)/2 \times 1 (2400)}$$

$$(2.7183)^{-0.3} \leqslant R > (2.7183)^{-0.005}$$

Confidence interval:

$$0.74 \leqslant R > 0.995$$

b. <u>Binomial type data.</u> — A sample of 20 items was taken from a lot of 100 components and tested under the use condition. No failures were obtained.

To accept this specific lot the lower limit of the 90 percent one sided confidence limit for the reliability should be taken from the tables in Appendix 3H which are based on the hypergeometric distribution. The value found in these tables is 9 defectives in the original lot of 100 items. From this then the lower limit for the true reliability of the lot is:

$$R \text{ (lower limit)} = 1 - 9/100 = 0.91$$

On the assumption that this value is acceptable and the lot is placed in the stockpile for further testing, the reliability of the items that this lot represents should now be determined from the tables in Appendix 3B which are based on the binomial distribution. From these tables, the lower limit of the true reliability of the items the lot represents is:

$$R \text{ (lower limit)} = 1.000 - 0.109 = 0.881$$

c. <u>Systems:</u> (1) A group of 10 telemetered missiles were flight tested. The number of failures found in each missile is as follows:

Missile number	Number of failures
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	3
9	0
10	0
Total	3

The point estimate for reliability as the probability of no failures under the test condition can be calculated as follows:

$$R = e^{-\overline{X}}$$

When: $\overline{X} = 3/10$

Point estimate:

$$R = (2.7183)^{-0.3} = 0.74$$

The 90 percent two sided confidence interval for the true reliability (R) can be calculated as follows:

$$e^{-U/n} \leqslant R \leqslant e^{-L/n}$$

When:

e = 2.7183

U = 7.75 (from Appendix 3D).

L = 0.818 (from Appendix 3D).

n = 10

$$(2.7183)^{-7.75/10} \leqslant R \leqslant (2.7183)^{-0.818/10}$$

Confidence Interval:

$$0.46 < R \leqslant 0.92$$

The point estimate and 90 percent two sided confidence interval calculated from the above example, using the binomial distribution is:

Point estimate:

$$R = 1 - 1/10 = 0.90$$

Lower limit (for defectives):

$$p_1 = \frac{a}{a + (n - a + 1) F_1}$$

When:

a = 1 (number of defective systems)

n = 10 (number of systems)

60

Degrees of freedom:

$$V_1 = 2 (10 - 1 + 1) = 20$$

$$V_2 = 2 \times 1 = 2$$

$F_1 = 19.4$ (from Appendix 3E Table 2B)

$$p_1 = \frac{1}{1 + 10 \times 19.4} = 0.0051$$

Upper Limit (for defectives)

$$p_2 = \frac{(a + 1) F_2}{(n - a) + (a + 1) F_2}$$

When:

a   =   1 (number of defective systems)

n   =   10 (number of systems)

Degrees of freedom:

$$V_1 = 2 (1 + 1) = 4$$

$$V_2 = 2 (10 - 1) = 18$$

$$F_2 = 2.93 \text{ (from Appendix 3E table 2A)}$$

$$p_2 = \frac{2 \times 2.93}{9 + 2 \times 2.93} = 0.396$$

Confidence interval:

$$1 = (0.396) = 0.604 \leqslant R \leqslant 1 - (0.0051) = 0.9949$$

(2) A system's reliability was calculated from a total of 200 test specimens and found to be $\hat{R} = 0.995$. The 95 percent one sided lower confidence limit is:

Upper Limit (for defectives):

$$p_2 = \frac{(a+1) F_2}{(n-a) + (a+1) F_2}$$

When:

a = 0.005 x 200 = 1 (average number of defectives)

n = 200

Degrees of freedom

$V_1 = 2 (1 + 1) = 4$

$V_2 = 2 (200-1) = 398 \approx \infty$

$F_2 = 2.37$ (From Appendix 3E Table 2A)

$$p_2 = \frac{2 \times 2.37}{(200-1) + 2 \times 2.37} = 0.0233$$

Lower confidence limit:

$$R = 0.995 - 0.023 = 0.972$$

B. Factorial Experiment

This example demonstrates how factorially designed environmental experiments can be used in combination with tests of increased severity. A simple three-treatment-experiment example is given below. The treatments used in this example are identified and defined as follows:

Identification	Treatment
A	Transportation vibration
B	Flight shock
C	High temperature

For purposes of the factorial design, each treatment is considered to have two levels:

    a.   Lower level is the absence of the treatment (designated by subscript 1).

    b.   Higher level is the presence of the treatment (designated by subscript 2).

The total number of possible combinations of three treatments, each at two levels, is two cubed or 8. These 8 combinations can be written in the following pattern:

	A_1		A_2	
	B_1	B_2	B_1	B_2
C_1	(1)	b	a	(a + b)
C_2	c	(b + c)	(a + c)	(a + b + c)

A minimum of 8 items would be required for this plan, each receiving different treatment combinations as follows:

Item Number	Treatment combinations
1	None (1)
2	B only
3	A only
4	A + B
5	C only
6	B + C
7	A + C
8	A + B + C

By using the letters (a, b, and c) and symbol (1) to represent the results obtained from testing the eight items, it can be shown symbolically that the treatment effects can be independently determined, using the total number of items in the entire experiment for each treatment as follows:

<u>Effect of treatment A</u>

$$a + (a + b) + (a + c) + (a + b + c) -$$

$$[(1) + b + c + (b + c)] = 4A$$

Effect of treatment B

$$b + (b + c) + (a + b) + (a + b + c) -$$

$$[(1) + c + a + (a + c)] = 4B$$

Effect of treatment C

$$c + (b + c) + (a + c) + (a + b + c) -$$

$$[(1) + b + a + (a + b)] = 4C$$

One-fourth of these differences equals the average effect of the respective treatments. From the above equations it can be seen that the results obtained from the eight items have been used three times — once for each treatment. This procedure produces an effective sample size equal to 3 x 8, or 24 items. Each treatment effect has been determined independently of the others with a precision equal to the total number of items used in the experiment.

The above three-factor factorial can be used as an example of a fractional factorial design as follows:

	A_1		A_2	
	B_1	B_2	B_1	B_2
C_1	-	b	a	-
C_2	c	·	·	(a + b + c)

A minimum of four items is required in this design. As before, the separate effects can be determined by a process of summation and subtraction as follows:

Effect of treatment A

$$a + (a + b + c) - (b + c) = 2A$$

Effect of treatment B

$$b + (a + b + c) - (a + c) = 2B$$

Effect of treatment C

$$c + (a + b + c) - (a + b) = 2C$$

One-half of these differences equals the average effect of the respective treatments.

When there is only one item available for each treatment combination, and only success and failure data are available, the usual analysis of variance cannot be used but the remaining advantages of the factorial design (given previously) still pertain. The above differences, which will be binomial proportions in this case, can be compared by the Fisher exact method for 2 x 2 contingency tables (ref. 7) to determine the treatment effects. A very convenient set of tables for this purpose can be found in ref. 8,* which contains tables of minimum contrasts based on Fisher's exact method.

      a. <u>Sample calculations.</u> The full three-factor-experiment used above might give the following typical set of results, when the figure "one" is entered as a "failure" and a "zero" is entered as a "success." It is assumed that a knowledge of the item being tested has led to the decision that transportation vibration, flight shock, and high temperature *in that order,* are the three environmental conditions most likely to affect the important functioning characteristic of this item; this characteristic is waterproofness. The treatment procedure and worksheet (to record results) for this experiment would be the following two-entry table. A plus mark in the item column means that the item received the corresponding treatment, while a "blank" means that the item did not receive the treatment.

Treatment procedure

Order of Treatment	Item Number							
	1-4	5-8	9-12	13-16	17-20	21-24	25-28	29-32
Transportation vibration (A)			+	+			+	+
Flight shock (B)		+		+		+		+
High temperature (C)					+	+	+	+
Results: Replication 1	1	0	0	1	0	0	1	1
2	0	0	0	0	0	1	1	1
3	0	1	0	1	0	1	1	1
4	1	1	0	1	0	0	1	1
Totals	2	2	0	3	0	2	4	4

The results of one complete replication should be obtained under a single set of controlled conditions (e.g., in the same day, same operators, same instruments, etc.), before going to the next replication. This will make it possible to determine whether conditions changed significantly during the experiment.

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*"See also Appendix 3A"

Placing these results in the usual factorial matrix, the following table would be obtained:

	A_1		A_2	
	B_1	B_2	B_1	B_2
C_1	1	0	0	1
	0	0	0	0
	0	1	0	1
	1	1	0	1
	2	2	0	3
C_2	0	0	1	1
	0	1	1	1
	0	1	1	1
	0	0	1	1
	0	2	4	4

In preparation for analyzing these results, the usual summing process would give the following series of two-factor tables:

Summing over A:

	B_1	B_2	Row Totals
C_1	2	5	7
C_2	4	6	10
Column totals	6	11	17

Summing over B:

	A_1	A_2	Row Totals
C_1	4	3	7
C_2	2	8	10
Column Totals	6	11	17

Summing Over C:

	A_1	A_2	Row Totals
B_1	2	4	6
B_2	4	7	11
Column Totals	6	11	17

Note that approximately 50 percent (17/32) failures were obtained. This is the condition under which the greatest resolution of effects is obtained. Each one of the marginal totals is the sum of 16 observations. The results can now be analyzed and interpreted as follows:

Source	Effects	Test of Significance*
Main Effects		
Transportation vibration (A)	6/16 vs 11/16	Non-significant
Flight shock (B)	6/16 vs 11/16	Non-significant
High temperature (C)	7/16 vs 10/16	Non-significant
Replication	1. 4/8	Non-significant
	2. 3/3	
	3. 5/8	
	4. 5/8	

*From Appendix 3A Table 1.

Source	Effects	Test of Significance*
Interactions		
A x B	8/16 vs 9/16	Non-significant
A x C	5/16 vs 12/16	Significant
B x C	8/16 vs 9/16	Non-significant
A x B x C	6/16 vs 11/16	Non-significant

*From Appendix 3A Table 1.

b. Interpretation (when the above order is used)

(1) The replication effect is not significant. This means that the conditions of the experiment did not change significantly from the beginning to the end. Therefore, the results can be accepted as valid from this standpoint.

(2) None of the effects is significant except the A x C interaction. This means that the combination of transportation vibration and high temperature treatments has caused a larger difference in the number of failures than would be expected due to chance variations alone.

(3) None of the treatments taken alone is significant, although the flight shock and transportation vibration effects approach significance. These results suggest the need for additional flight-shock and transportation vibration tests if these treatments are considered important from an engineering point of view.

These results show clearly that the combination of transportation vibration and high temperature is the most severe condition. From this, reliability should be defined in terms of waterproofness after transportation vibration and high temperature. If this reliability is acceptable, the waterproofness reliabilities under all of the other conditions used will also be acceptable.

C. Tests of Increased Severity

a. Bruceton method (ref. 3). The results from the factorial experiment described above show that the Bruceton "up-and-down" procedure can be conducted by varying the severity (g-force level) of the transportation vibration treatment and using the same high temperature (without variation) as that used in the factorial experiment. This can be done since the high temperature main effect (difference in the number of failures between the presence and absence of this treatment) is not significant. Assuming that the average g-force expected in use is 4 g's with a standard deviation of 2 g's, then using increments of 2 g's and starting at 6 g's, apply the vibration and temperature treatments and conduct the waterproofness test on one new, unused item. If the item does not fail, increase the g-level one increment and again test one new, unused item. Continue this process of increasing the g-level one increment at a time and testing one new, unused item at each g-level until the first failure is obtained. Then reverse the process by decreasing the g-level one increment at a time and testing one new, unused item at each g-level until an item successfully passes the

waterproofness test. Repeat the process of increasing the g-level to failure and decreasing the g-level to success, until at least 12 items have been made to fail in this manner.

Ordinarily the Bruceton method would not be used for this test. Since the result of each test must be known before the next test can be run, this method would consume far too much time. It is used here for demonstration purposes only. In practice the Two-stimuli method should be used for a test of this kind, since several of the tests could be conducted concurrently with a considerable saving of time.

Record the results in graphic form for convenience and count the number of failures obtained at each g-level. The following can be used as an example of the type of observed data that could be obtained:

Diagram Number I	Distribution of fail ..s

g-level	Distribution
22	1
20	2
18	5
16	3
14	1
	Total 12

where:

+ = Success.
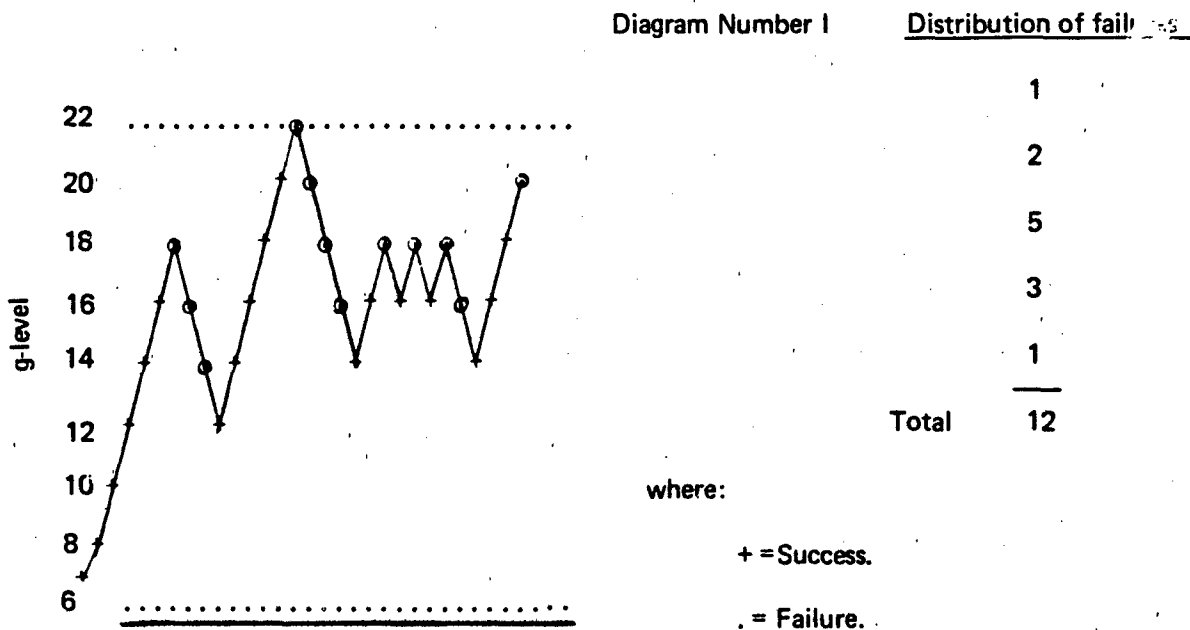
. = Failure.

Calculate the average ($\overline{X}$) and standard deviation (s) of the failure rate as follows (ref. 3):

Where:

$$A = \sum_{x=o}^{x=k} fx; \qquad B = \sum_{x=o}^{x=k} f(x)^2$$

k = Number of g-levels over which the failures are distributed.

f = Observed frequency of failure.

x = Code numbers used for ease of calculation.

$$X = y^1 + d(A/N - 1/2)$$

$$S = 1.62 d \left(\frac{NB - A^2}{N^2} + .029 \right)$$

Where:

y^1 = Lowest level at which a failure is obtained.

d = 2 g's. — the increment used.

N = Total number of failures.

This formula for the standard deviation is an approximation which is quite accurate when $\frac{(NB - A^2)}{N}$ exceeds 0.3.

Using these formulas and the observed data, the average and standard deviation for the above example can be calculated as follows:

f	x	fx	fx^2
1	4	4	16
2	3	6	18
5	2	10	20
3	1	3	3
1	0.	0	0
N = 12		A = 23	B = 57

$$\overline{X}_2 = 14 + 2(23/12 - 1/2) = 16.8 \text{ g's}$$

$$s_2 = 1.62 \times 2 \left[\frac{12 \times 57 - (23)}{(12)^2} + 0.029 \right] = 3.58 \text{ g's}$$

The cumulative frequency of the observed failure distribution plotted on linear probability paper closely approximates a straight line. From this it can be concluded that the assumption of normality is sufficiently valid for use as a basis to predict the expected reliability-in-use.

Therefore, the point estimate for the (waterproofness) reliability-in-use can be calculated as follows:

$$\hat{R} = 1 - P$$

Where:

P = The probability of failure-in-use.

The probability of failure-in-use can be measured by the area under the normal curve associated with the Z-value calculated as follows:

$$Z = \frac{(X_1 - X_2) - (M_1 - M_2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

When: $X_1 \geqslant X_2$ a failure is obtained.

$$Z \geqslant \frac{M_2 - M_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

Where:

X_1 = Any stress value.

X_2 = Any strength value.

M_1 = True mean of the stress distribution.

M_2 = True mean of the strength distribution.

σ_1^2 = True variance of the stress distribution.

σ_2^2 = True variance of the strength distribution.

$$\frac{M_2 - M_1}{M_1} = \text{Safety margin.}$$

The relation between the safety margin and a measure of probability is shown above. If the product of the safety margin and the average stress in use is divided by $\sqrt{\sigma_1^2 + \sigma_2^2}$, we have a measure of probability — the standard deviate.

Graphically, the relationship between the test conditions and use condition can be depicted as follows:

Diagram Number II

| Use condition | Test condition |
| Stress | Strength |

$s_2 = \dfrac{x}{y}$
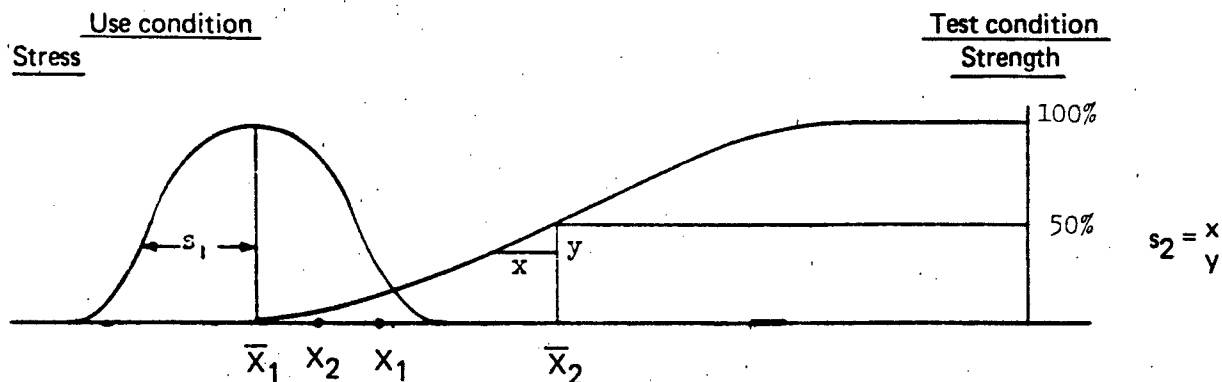
Where:

The horizontal axis represents the g-forces increasing to the right. The bell shaped curve represents the distribution of g-forces under the use condition (the stress curve).

The S-shaped curve represents the distribution of failures under the test condition obtained by the Bruceton method (the strength curve).

When:

$\overline{X}_1$ = 4 g's - an estimate of M_1 the true average stress.

s_1 = 2 g's - an estimate of σ_1 the true standard deviation of the stress.

$\overline{X}_2$ = 16.8 g's - an estimate of M_1 the true average strength (the 50 percent point on the strength curve.)

s_2 = 3.48 g's - an estimate of σ_2, the true standard deviation of the strength.

X_1 = any stress value.

X_2 = any strength value.

From the above, the *average* (point estimate) reliability-in-use can be calculated as follows:

$$T \geqslant \frac{\overline{X}_2 - \overline{X}_1}{\sqrt{s_1^2 + s_2^2}}$$

$$T \geqslant \frac{16.8 - 4.0}{\sqrt{(3.58)^2 + (2)^2}} \geqslant 3.12$$

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From Appendix 3G the probability of a failure-in-use associated with this T-value is: $P_1 < 0.00090$.

Reliability Point Estimate:

$$\hat{R} \geq 1 - 0.00090 \geq 0.99910$$

The lower confidence limit for the one sided 95 percent confidence level can be calculated as follows:

The 95 percent single sided lower confidence limit of the average strength:

$$\overline{X}_2 - \frac{ts_2}{\sqrt{n_2}} \quad \text{(see ref. 3)}$$

Where:

$\overline{X}_2 = 16.8$ - the observed average strength.

$t = 1.80$ — the t-value associated with the single sided 95 percent confidence level and eleven (n_2-1) degrees of freedom found in Appendix 3F.

$s_2 = 3.58$ — the standard deviation of the strength.

$n_2 = 12$ — the sample size used to determine the strength.

Then the 95 percent single sided lower confidence limit for the strength is:

$$16.8 - \frac{(1.80)\,(3.58)}{\sqrt{12}} = 14.94$$

The lower bound of the reliability-in-use can be calculated as follows:

$$T_2 \geq \frac{14.94 - 4.0}{\sqrt{(3.58)^2 + (2)^2}}$$

$$T_2 > 2.67$$

The probability of a failure-in-use associated with this T-value (appendix 3G) is:

$$\hat{P} \leq 0.0038$$

The lower bound of the reliability-in-use is:

$$R \geq 1 - 0.0038 \geq 0.9962$$

These values are the predicted reliabilities-in-use. They have been demonstrated with a total of 62 items (32 in the factorial experiment and 30 in the Bruceton up-and-down method). To demonstrate this reliability by doing all of the testing at the use condition, would require that 786 items be tested _without_ a single failure.

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b. <u>Two stimuli method</u>. The results of the factorial experiment described above can also be used to exemplify this method of predicting reliability-in-use. Beginning at one increment (one standard deviation) above the average use-condition, the g-level can be increased one increment at a time (as in the Bruceton method) until the first failure is obtained. The proportions of failures at this point and at a point three increments above this point are as follows:

g-level	Proportions of failures
12.0	1/10
18.0	6/10

This method (under these conditions) required a total of 23 test specimens — 3 before obtaining the first failure and 10 at each of the two points on the curve.

Calculate the average ($\overline{H}$) and the standard deviation (S) of the failure rate as follows (ref. 10): $\overline{H} = X_1 + d\,(\overline{H})'$

$$S = d\,S^1$$

Where:

X_1 = 12 g's the weaker stimulus.

p_1 = 0.10, the proportion of failures at 12 g's.

p_2 = 0.60, the proportion of failures at 18 g's.

d = 6 g's (18 — 12 g's) the increment used.

$\overline{H}'$ = 0.8350, the factor associated with p_1 and p_2 in table II of ref. 10 for calculating the average.

S^1 = 0.6515, the factor associated with p_1 and p_2 in table II of ref. 10 for calculating the standard deviation.

$\overline{H}$ = Average (50 percent point) of the failure distribution (strength) curve.

S = Standard deviation of the failure distribution (strength) curve.

When:

$\overline{H}$ = 12.0 + 6.0 (0.8350) = 17.0 g's.

S = (0.6515) = 3.9 g's.

74

These same values can be obtained graphically by simply plotting the proportion of failures versus the corresponding g-forces on linear probability paper. The average is the g-force corresponding to the 50 percent point, and the standard deviation is the slope of the line thru the two points, or is the difference between the g-forces corresponding to the 16 and 50 percent points.

Using these values for the average and standard deviation, the point estimate and lower confidence limit for the (waterproofness) reliability-in-use can be calculated as in the examples for the Bruceton method.

APPENDIX 2

GLOSSARY OF TERMS

Attribute

A qualitative characteristic (such as acceptable or rejectionable, success or failure, rusted or not rusted, wet or dry, black or white, miss or hit), which can have two or more categories.

Attribute data

Data denoting a qualitative characteristic. This type of data can have only discrete values and is derived by counting the number of times each category occurs, such as, four failures and six successes.

Best Estimate

An estimator is said to give the "best estimate" of the true population parameter if it complies with the following requirements which are taken as the definition of the word "best":

A. The average of all possible values of the estimator equals the true population parameter.

B. In any particular case the deviation of the estimator from the true population parameter is less than any other possible estimator.

Binomial data

Attribute data that has only two categories or only two possible outcomes such as success and failure.

Blocking

In experimental design, a block is a homogeneous group of items, all treated under controlled conditions such as by the same operator, the same calibration of the measuring instrument, or the same short period of time. The purpose of blocking is to reduce the effect of the heterogeneity of material and changing conditions by dividing the experiment into rational subdivisions.

Confidence interval

The range of values within which the true population parameter (mean or standard deviation) is expected to lie. The confidence level associated with this interval is a probability statement expressing the proportion of the time the true value is expected to be within the interval.

Confidence level

The confidence level is the probability of being right in our predictions or conclusions. This value is equal to one minus the error of the first kind. The magnitude of this error that can be tolerated should be established during the planning stage of the experiment (prior to data collecting) based on the consequences of being wrong and thereby establish the confidence level.

Confounding

When certain comparisons can be made only for treatments in combination and not for separate treatments, those treatment effects are said to be confounded. Conclusions drawn about the separate effects in this case will be ambiguous. Confounding is often a deliberate feature of the experimental design but may arise from inadvertent imperfections.

Criterion

The measurable characteristic used to evaluate the treatment effects. Criteria can also be considered as the dependent variables used as a standard of reference to distinguish between the independent variable effects. Velocity, functioning time, voltage, rate of detonation, etc., can be criteria.

Degrees of freedom

The number of degrees of freedom is equal to the number of independent observations minus the number of parameters (such as the mean) estimated. That is, degrees of freedom usually equal the sample size minus one. In computing the variance for example, only (n-1) of the deviations from the mean can be independent. The nth deviation has to be restricted in order to make the sum of all "n" deviation total zero.

Effect

In statistics the meaning of the word effect is synonomous with the word difference. A treatment effect is the difference caused by the treatment, such as the difference in the measured results before and after the treatment.

Efficiency

An estimator or an experimental design is said to be efficient if a given precision can be obtained with a smaller sample size or with less time and cost.

Error

Chance variations are considered errors in statistics. Deviations from the expected value, due to chance, form the familiar bell shaped normal curve. This is sometimes called the normal curve of error. Error in the statistical sense does not imply that a mistake has been made.

Error mean square

The error mean square is the variance and is also the square of the standard deviation. It is calculated by finding the sum of the squares of the deviations of the individual sample values from their mean and dividing by the number of degrees of freedom.

Error of estimate

The difference between an estimated value and the true value.

Error of first kind

If, as a result of a statistical test, the null hypothesis is rejected when it is true, then it is said that an error of the first kind is committed. This type of error is also called:

a. The alpha error.

b. The producer's risk.

c. The risk of rejecting good material.

The magnitude of this error should be established from the consequences of being wrong and controlled at that level by calculating the required sample size.

Error of observation

An error of observation arises from imperfections in the method of measurement or from human mistakes.

Error of second kind

If, as a result of a statistical test, the null hypothesis is accepted when it is false, then it is said that an error of the second kind is committed. This type of error is also called:

a. The beta error.

b. The consumer's risk.

c. The risk of accepting poor material.

After the error of the first kind has been established, the error of the second kind is controlled by the sample size. This error is very important in Ordnance work because it controls the probability of accepting poor material.

Estimate

An estimate is the particular value obtained by an estimator in a given set of circumstances.

Estimator

An estimator is the method of estimating a constant of a parent population. It is usually expressed as a function of sample values (such as the average) and therefore is a variable.

Experimental error

Experimental error is the chance variation to be expected under controlled conditions. It is not the result of mistakes in experimental design or avoidable imperfections in technique.

Experimental unit

An experimental unit is the smallest subdivision of the experimental material that can receive different treatments in the actual experiment. It is also known as a test specimen.

Factor

A factor is a quantity under examination (in an experiment) as a possible cause of variation. In practice the terms factor, treatment, and variable are loosely used interchangeably in this sense.

Factorial experiment

An experiment which investigates all of the possible treatment combinations that may be formed from the factor versions under investigation.

Fractional factorial experiment

This is a fractional part of a factorial experiment. When three or more factors are used in a factorial experiment only a fractional part (1/2, 1/4, 1/8) of the total number of possible combinations need be used if certain of the interactions can be considered negligible. This device can be resorted to without loss of efficiency when the number of factors to be investigated makes the full factorial so large that it is impractical to use.

Hypothesis

A hypothesis is a contention based on preliminary observation of what appears to be fact. It is the prediction derived from past experience that is to be verified or rejected by experimentation. Natural "laws" are hypothesis which have been subjected to various tests and have been accepted. In statistical tests two hypothesis are used:

a. The _null hypothesis_ is a hypothesis of "no difference." This is the assumption that the contemplated changes will make no difference. This hypothesis is formulated for the express purpose of being rejected in the process of controlling the error of the first kind.

b. The _alternative hypothesis_ is the operational statement of the experimenter's prediction. It is the positive statement that the changes will make a detectable difference. If the resultant data reject the null hypothesis the alternative hypothesis will be accepted.

Independence

Measurements are independent if the taking of one does not effect any of the others. That is, there is no correlation among them. Treatment effects are said to be independent if, in an orthogonal experiment, there is no interaction.

Interaction

Interaction is a measure of the extent to which the effect of changing the level of one factor depends on the level of another factor. Interaction is said to be present when a certain particular combination of treatments produces unusual (unpredictable) results. Only factorial type experiments can measure interaction effects.

Levels

The level of a factor (or treatment) denotes the intensity with which it is used or applied. Levels of a factor may be either qualitative, such as presence and absence of the treatment, or the levels may be quantitative, such as the number of volts applied.

Main effects

A main effect is the average difference(s) between (or among) the levels of a variable or treatment when averaged over all of the other treatmens which form a part of the same orthogonal experiment. If significant interaction effects are present, care must be taken in stating the main effects. In such cases the level of the interacting treatment associated with the stated main effect must also be stated.

Normal distribution

The physical appearance of a normal distribution is the familiar bell-shaped curve. A normal distribution can not be represented by only a single curve. It is actually a family of curves whose areas under them are distributed in a very specific manner. A normal curve has the following properties:

a. Continuous.

b. Symmetrical.

c. Unimodal.

d. Asymptotic to x-axis.

e. Completely described by the mean and standard deviation.

f. The distance between the ordinate of the mean and the inflection point on either half of the curve is equal to the standard deviation.

g. The area included between the ordinates drawn thru the two inflection points equals 68.27 percent of the total area under the curve.

Parameter

A parameter is a quantity such as the mean or standard deviation, calculated from a population. The population mean and standard deviation are parameters and as such are constants. In actual practice parameters are usually unknown.

Point estimate

This is one of the two principal bases of estimation in statistical analysis. Point estimation endeavors to give the best single estimated value of a parameter, as compared with interval estimation which specifies a range of values. Since a point estimate includes an error of measurement, the difference between a point and an interval is not always clear. In interpretation they often amount to the same thing.

Population

A population is any set of individuals or objects having some common observable characteristic. The term population may refer either to the individuals measured or to the measurements themselves. A population is usually considered to consist of an *infinite* number of individuals. The curve of the normal distribution graphically represents a population.

Precision

Precision is a property of the measuring system and refers to the ability of the system to reproduce previous results. Precision should be distinguished from accuracy which refers to the magnitude of the difference between the observed values and the true value of the characteristic being measured. Precision should also be distinguished from the sensitivity of the measuring system which is the ability of the system to detect actual variations that occur. An insensitive system will give the false impression of high precision (small variation).

Probability

In applied statistics probability can be considered a relative frequency or a simple proportion. Probability is the relative frequency of events in a very long sequence of trials. For example, the probability of a particular coin falling heads up is the ratio of the number of heads occurring to the total number of trials in a sequence of trials. In somewhat similar fashion a normal distribution can be formed from a very large body of data. As a result, the area under the normal curve is used as a measure of probability.

Randomization

The word randomization has a very special technical meaning in statistics. It means rearranging a group of items or numbers into a series or sequence having no recognizable pattern. The essential feature of randomization is that is should be an objective impersonal procedure. Whether or not proper randomization has been obtained should not be determined by an examination of the individuals randomized, but rather by examining the properties of the procedure by which randomization was accomplished. The objectives of randomizing are as follows:

a. To give the laws of chance free play.

b. To give every possible sequence an equally likely chance of occurring.

c. To assure that adjacent individuals are completely independent.

d. To remove biases of any kind.

e. To prevent systematic error.

Reliability

In missile technology reliability is the probability of success in performing a specified function, under a specified condition, for a specified length of time, and after a specified period of time. From this it is clear that any particular component can have many reliability values simultaneously-one for every possible combination of function, condition, and time.

Replication

Replication is the performance of an experiment in its entirety one or more times. Two or more replications are usually for the purpose of obtaining an independent measure of the sampling or experimental error Replication should be distinguished from repetition, in that, replication means repetition carried out under the same conditions, at the same time, by the same operators, with the same instruments, and with the same homogeneous material. A replication is sometimes considered a block.

Sample

Any finite subset of a population is a sample of that population.

Sample size

The sample size is the number of items or individual values in the sample.

Standard deviation

a. Definition. The standard deviation is a measure of the variation among the individual values in a sample and a measure of the dispersion among the individual values in a frequency distribution. It is the most efficient measure of precision and is designated by the lower case letter "s". This value is large for large variations (poor precision) and small for small variations (good precision). Although the word "error" is sometimes used in referring to the standard deviation or its square (the variance) these values can measure only precision in the true sense of the word. They do not measure accuracy.

If the term standard deviation is stated alone and not modified or otherwise qualified by an accompanying word or phrase, it is understood that the term refers to the standard deviation of the individual sample measurements. This value can be calculated from the sample data and is a *variable*.

There are two additional kinds of standard deviations:

1.   The *population standard deviation* which is a *constant* and cannot be calculated from the sample data. This value is designated by the small Greek letter sigma and is usually considered unknown unless a very large body of data is collected to measure it or unless it is assigned a value as in a specification requirement.

2.   The *standard deviation of the mean* is a measure of the variation among several sample averages. This value can be calculated from sample data and it is a variable. It is usually designated by the lower case letter "s" with the subscript X. If all of the sample sizes are equal this value can be calculated by dividing the standard deviation of the individual sample values by the square root of the number of individual values in each of the samples.

b.   Calculation of the standard deviation for variable type data:

$$s = \sqrt{\frac{\sum\limits_{i=1}^{i=n}(\overline{X}-x_i)^2}{n-1}} = \sqrt{\frac{\sum\limits_{i=1}^{i=n}(x_i)^2 - \left(\sum\limits_{i=1}^{i=n}x_i\right)^2}{\frac{n}{n-1}}}$$

where:

s = Sample standard deviation of the individual values.

$\sum\limits_{i=1}^{i=n}$ = This symbol means to add all of the "n" quantities designated by the parentheses. It is read: sum from i = 1 to i = n.

$\overline{X}$ = Sample average.

x_i = Any one of the "n" values that make up the sample.

n = Sample size or the number of individual values that make up the sample.

(n-1) = Number of degrees of freedom associated with the standard deviation.

s^2 = Sample variance of the individual values.

$s/\sqrt{n}$ = Sample standard deviation of the mean (s_x).

Statistic

A statistic is a summary value calculated from a sample of values. The sample mean is a statistic and as such is a variable, not a constant.

Statistical significance

A difference or an effect is said to be statistically significant if it is greater than that expected due to chance alone. If the probability (chance) is very small that a value came from a particular population, the difference between that value and the mean of the population is said to be statistically significant.

Statistics

The subject of statistics is the science of collecting, analyzing, and interpreting numerical data.

Treatment

In experimentation, a treatment is a stimulus which is applied in order to observe the effect on the experimental situation. A treatment may refer to a physical substance, a procedure, or anything which is capable of controlled application. In statistical parlance a treatment is the variable being studied or the experimental condition.

True Value

The true value is another expression for a population parameter such as the population mean or standard deviation. The true value can also be the expected value or the theoretical value.

Validation

Validation is a procedure which provides, by reference to independent sources, evidence that an inquiry is free from bias, or otherwise conforms to its declared purpose. In statistics it is usually applied to a sample investigation with the object of showing that the sample is reasonably representative of the population and that the information collected is accurate.

Variable

A variable is any quantity or measurable characteristic which varies. More precisely in statistics a variable is any quantity which can have any one of a specified set of values.

Variable data

Variable data is a term used to describe a type of data that can vary on a continuous scale from zero to infinity. Weight in pounds, length in feet, E.M.F. in volts, and temperature in degrees are variable type data.

Variance

Variance is a measure of variation in a sample, or dispersion in a frequency distribution. The variance is equal to the square of the standard deviation.

APPENDIX 3A

Table 1

MINIMUM CONTRASTS 95% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/No. of A's in Sample (2)
4	0/4 1/-
5	0/4 1/5 2/-
6	0/5 1/6 2/-
7	0/5 1/6 2/7 3/-
8	0/5 1/6 2/7 3/8 4/-
9	0/5 1/6 2/8 3/8 4/9 5/-
10	0/5 1/7 2/8 3/9 4/10 5/10 6/-
11	0/5 1/7 2/8 3/9 4/10 5/11 6/11 7/-
12	0/5 1/7 2/8 3/9 4/10 5/11 6/12 7/12 8/-
13	0/5 1/7 2/8 3/9 4/10 5/11 6/12 7/13 8/13 9/-
14	0/5 1/7 2/8 3/10 4/11 5/12 6/12 7/13 8/14 9/14 10/-
15	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/14 9/15 10/15 11/-
16	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15 9/15 10/16 11/16 12/-
17	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15 9/16 10/16 11/17 12/17 13/-
18	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15 9/16 10/17 11/17 12/18 13/18 14/-
19	0/5 1/7 2/9 3/10 4/11 5/12 6/14 7/14 8/15 9/16 10/17 11/18 12/18 13/19 14/19 15/-

Table 1 (Continued)

MINIMUM CONTRASTS 95% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/No. of A's in Sample (2)
20	0/5 1/7 2/9 3/10 4/11 5/13 6/14 7/15 8/16 9/16 10/17 11/18 12/19 13/19 14/20 15/20 16/-
30	0/6 1/8 2/9 3/11 4/12 5/13 6/15 7/16 8/17 9/18 10/19 16/25 17/25 20/28 21/28 23/30 24/30 25/-
40	0/6 1/8 2/9 3/11 4/12 5/14 6/15 7/16 8/18 9/19 10/20 23/33 24/33 27/36 28/36 30/38 31/38 33/40 34/40 35/-
50	0/6 1/8 2/10 3/11 4/13 5/14 6/15 7/17 8/18 9/19 10/20 11/22 29/40 30/40 34/44 35/44 38/47 39/47 41/49 42/49 44/50 45/-
60	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/18 9/20 10/21 11/22 12/23 13/24 14/26 35/47 36/47 41/52 42/52 45/55 46/55 48/57 49/57 51/59 52/59 53/60 54/60 55/-
70	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/18 9/20 10/21 11/22 12/23 13/25 18/30 19/32 20/33 39/52 40/52 46/58 47/58 51/62 52/62 55/65 56/65 58/67 59/67 61/69 62/69 63/70 64/70 65/-
80	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/19 9/20 10/21 11/22 12/24 13/25 14/26 15/27 16/29 23/36 24/38 43/57 44/57 52/65 53/65 57/69 58/69 62/73 63/73 65/75 66/75 68/77 69/77 71/79 72/79 73/80 74/80 75/-

MINIMUM CONTRASTS 95% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/ No. of A's in Sample (2)
90	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/19 9/20 10/21 11/23 12/24
	13/25 14/26 15/28 20/33 21/35 31/45 32/47 44/59 45/59 56/70 57/70
	63/76 64/76 68/80 69/80 72/83 73/83 75/85 76/85 78/87 79/87 81/89
	82/89 83/90 84/90 85/-
100	0/6 1/8 2/10 3/11 4/13 5/15 6/16 7/17 8/19 9/20 10/21 11/23 12/24
	13/25 14/27 18/31 19/33 25/39 26/41 60/75 61/75 68/82 69/82 74/87
	75/87 78/90 79/90 82/93 83/93 86/96 87/96 88/97 89/97 91/99 92/99
	93/100 94/100 95/-
150	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/20 10/22 11/23 12/24
	13/26 14/27 15/28 16/30 19/33 20/35 25/40 26/42 32/48 33/50 41/58
	42/60 91/109 92/109 101/118 102/118 109/125 110/125 116/131 117/131
	121/135 122/135 125/138 126/138 129/141 130/141 133/144 134/144
	136/146 137/146 139/148 140/148 141/149 142/149 143/150 144/150 145/-
200	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/21 10/22 11/23 12/25
	13/26 14/27 15/29 18/32 19/34 22/37 23/39 27/43 28/45 33/50 34/52
	41/59 42/61 51/70 52/72 65/85 66/87 114/135 115/135 129/149 130/149
	140/159 141/159 149/167 150/167 156/173 157/173 162/178 163/178
	167/182 168/182 172/186 173/186 176/189 177/189 180/192 181/192
	183/194 184/194 186/196 187/196 189/198 190/198 191/199 192/199
	193/200 194/200 195/-

APPENDIX 3A

Table 1 (Continued)

MINIMUM CONTRASTS 95% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/ No. of A's in Sample (2)
300	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/21 10/22 11/24 12/25
	13/26 14/28 15/29 16/30 17/31 18/33 19/34 20/35 21/37 24/40 25/42
	29/46 30/48 35/53 36/55 41/60 42/62 48/68 49/70 56/77 57/79 66/88
	67/90 78/101 79/103 95/119 96/121 180/205 181/205 198/222 199/222
	211/234 212/234 222/244 223/244 231/252 232/252 239/259 240/259
	246/265 247/265 253/271 254/271 259/276 260/276 264/280 265/280
	268/283 269/283 273/287 274/287 277/290 278/290 280/292 281/292
	283/294 284/294 286/296 287/296 289/298 290/298 291/299 292/299
	293/300 294/300 295/-

Table 2

MINIMUM CONTRASTS 99% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/ No. of A's in Sample (2)
5	0/5 1/-
6	0/6 1/-
7	0/6 1/7 2/-
8	0/6 1/8 2/8 3/-
9	0/6 1/8 2/9 3/9 4/-
10	0/7 1/8 2/9 3/10 4/-
11	0/7 1/8 2/9 3/10 4/11 5/-
12	0/7 1/8 2/10 3/11 4/11 5/12 6/-
13	0/7 1/9 5/13 6/13 7/-
14	0/7 1/9 6/14 7/14 8/-
15	0/7 1/9 7/15 8/15 9/-
16	0/7 1/9 2/10 3/12 4/13 5/14 6/14 8/16 9/16 10/-
17	0/7 1/9 2/11 7/16 8/16 9/17 10/17 11/-
18	0/7 1/9 2/11 8/17 9/17 10/18 11/18 12/-
19	0/7 1/9 2/11 9/18 10/18 11/19 12/19 13/-
20	0/7 1/9 2/11 4/13 5/15 6/16 7/16 10/19 11/19 12/20 13/20 14/-
30	0/8 1/10 2/12 3/13 4/15 10/21 16/27 17/27 18/28 19/29 20/29 21/30 22/30 23/-
40	0/8 1/10 2/12 3/14 4/15 5/17 8/20 9/22 19/32 20/32 24/36 25/36 27/38 28/38 29/39 30/39 31/40 32/40 33/-
50	0/8 1/10 2/12 3/14 4/15 5/17 6/18 7/20 9/22 10/24 27/41 28/41 31/44 32/44 34/46 35/46 37/48 38/48 39/49 40/49 41/50 42/50 43/-

Table 2 (Continued)

MINIMUM CONTRASTS 99% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRAILS IN EACH SAMPLE

N	No. of A's in Sample (1)/No. A's in Sample (2)
60	0/8 1/10 2/12 3/14 4/16 5/17 6/19 8/21 9/23 11/25 12/27 19/34 20/36 24/40 25/41 26/41 34/49 35/49 38/52 39/52 42/55 43/55 45/57 46/57 47/58 48/58 49/59 50/59 51/60 52/60 53/-
70	0/8 1/10 2/12 3/14 4/16 5/17 6/19 7/20 8/22 10/24 11/26 14/29 15/31 21/37 22/39 32/49 33/49 34/50 40/56 41/56 45/60 46/60 49/63 50/63 52/65 53/65 55/67 56/67 57/68 58/68 59/69 60/69 61/70 62/70 63/-
80	0/8 1/10 2/12 3/14 4/16 5/18 6/19 7/21 9/23 10/25 12/27 13/29 16/32 17/34 24/41 25/43 38/56 39/56 47/64 48/64 52/68 53/68 56/71 57/71 60/74 61/74 63/76 64/76 65/77 66/77 67/78 68/78 69/79 70/79 71/80 72/80 73/-
90	0/8 1/10 2/12 3/14 4/16 5/18 6/19 7/21 8/22 9/24 11/26 12/28 15/31 16/33 19/36 20/38 28/46 29/48 43/62 44/62 53/71 54/71 58/75 59/75 63/79 64/79 67/82 68/82 70/84 71/84 73/86 74/86 75/87 76/87 77/88 78/88 79/89 80/89 81/90 82/90 83/-
100	0/8 1/10 2/13 3/14 4/16 5/18 6/19 7/21 8/22 9/24 10/25 11/27 14/30 15/32 18/35 19/37 23/41 24/43 33/52 34/54 47/67 48/67 58/77 59/77 64/82 65/82 69/86 70/86 74/90 75/90 77/92 78/92 80/94 81/94 83/96 84/96 85/97 86/97 88/99 89/99 90/99 91/100 92/100 93/-

Table 2 (Continued)

MINIMUM CONTRASTS 99% (TWO SIDED) TEST

N = TOTAL NUMBER OF TRIALS IN EACH SAMPLE

N	No. of A's in Sample (1)/No. of A's in Sample (2)
150	0/8 1/11 2/13 3/15 4/16 5/18 6/20 7/21 8/23 9/24 10/26
	11/27 12/29 14/31 15/33 17/35 18/37 21/40 22/42 26/46
	27/48 31/52 32/54 39/61 40/63 51/74 52/76 75/99 76/99
	88/111 89/111 97/119 98/119 103/124 104/124 109/129 110/129
	114/133 115/133 118/136 119/136 122/139 123/139 125/141 126/141
	128/143 129/143 131/145 132/145 133/146 134/146 136/148 137/148
	138/149 139/149 140/150 141/150 142/150 143/-
200	0/8 1/11 2/13 3/15 4/16 5/18 6/20 7/21 8/23 9/24 10/26 11/27
	12/29 13/30 14/32 16/34 17/36 19/38 20/40 23/43 24/45 26/47
	27/49 31/53 32/55 36/59 37/61 43/67 44/69 51/76 52/78 63/89
	64/91 110/137 111/137 123/149 124/149 132/157 133/157 140/164
	141/164 146/169 147/169 152/174 153/174 156/177 157/177
	161/181 162/181 165/184 166/184 169/187 170/187 172/189 173/189
	175/191 176/191 178/193 179/193 181/195 182/195 183/196 184/196
	186/198 187/198 188/199 189/199 190/200 191/200 192/200 193/-
300	0/8 1/11 2/13 3/15 4/17 5/18 6/20 7/22 8/23 9/25 10/26 11/28
	12/29 13/31 15/33 16/35 17/36 18/38 20/40 21/42 23/44 24/46
	27/49 28/51 31/54 32/56 35/59 36/61 40/65 41/67 45/71 46/73 51/78
	52/80 58/86 59/88 66/95 67/97 76/106 77/108 88/119 89/121 107/139
	108/141 160/193 161/193 180/212 181/212 193/224 194/224 204/234
	205/234 213/242 214/242 221/249 222/249 228/255 229/255 234/260
	235/260 240/265 241/265 245/269 246/269 250/273 251/273 255/277
	256/277 259/280 260/280 263/283 264/283 266/285 267/285 270/288
	271/288 273/290 274/290 276/292

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
1	0		.500	.800	.900	.950	.990	.995
2	0		.129	.581	.684	.776	.900	.929
	1		.198	.912	.949	.975	.995	.995
3	0		.206	.430	.536	.632	.785	.829
	1		.500	.733	.804	.865	.941	.959
	2		793	.941	.966	.980	.997	.998
4	0		.159	.349	.438	.527	.684	.733
	1		.386	.604	.680	.751	.859	.889
	2		.614	.803	.857	.902	.958	.971
	3		.841	.948	.974	.987	.997	1.000
5	0		0.129	0.275	0.369	0.451	0.602	0.653
	1		0.314	0.490	0.584	0.657	0.778	0.815
	2		0.500	0.673	0.753	0.811	0.894	0.917
	3		0.686	0.831	0.888	0.924	0.967	0.977
6	0		0.109	0.235	0.319	0.393	0.536	0.586
	1		0.264	0.422	0.510	0.582	0.706	0.746
	2		0.421	0.585	0.667	0.729	0.827	0.856
	3		0.579	0.731	0.799	0.847	0.915	0.934
7	0		0.094	0.205	0.280	0.348	0.482	0.531
	1		0.228	0.371	0.453	0.521	0.643	0.685
	2		0.364	0.517	0.596	0.659	0.764	0.797
	3		0.500	0.650	0.721	0.775	0.858	0.882
	4		0.636	0.772	0.830	0.871	0.929	0.945
8	0		0.083	0.182	0.250	0.312	0.438	0.484
	1		0.201	0.330	0.406	0.471	0.590	0.632
	2		0.321	0.462	0.538	0.600	0.707	0.742
	3		0.440	0.584	0.655	0.711	0.802	0.830
	4		0.560	0.697	0.760	0.807	0.879	0.900

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
9	0		0.074	0.164	0.226	0.283	0.401	0.445
	1		0.180	0.298	0.368	0.429	0.544	0.585
	2		0.286	0.418	0.490	0.550	0.656	0.693
	3		0.393	0.529	0.599	0.655	0.750	0.781
	4		0.500	0.634	0.699	0.749	0.829	0.854
	5		0.607	0.732	0.790	0.831	0.895	0.913
10	0		0.067	0.149	0.206	0.259	0.369	0.411
	1		0.162	0.271	0.337	0.394	0.504	0.544
	2		0.259	0.381	0.450	0.507	0.612	0.648
	3		0.355	0.484	0.552	0.607	0.703	0.735
	4		0.452	0.581	0.646	0.696	0.782	0.809
	5		0.548	0.673	0.733	0.778	0.850	0.872
	6		0.645	0.761	0.812	0.850	0.907	0.923
11	0		0.061	0.136	0.189	0.238	0.342	0.382
	1		0.148	0.249	0.310	0.364	0.470	0.509
	2		0.235	0.350	0.415	0.470	0.572	0.608
	3		0.324	0.445	0.511	0.564	0.660	0.693
	4		0.412	0.536	0.599	0.650	0.738	0.767
	5		0.500	0.622	0.682	0.729	0.806	0.831
	6		0.588	0.705	0.759	0.800	0.866	0.886
12	0		0.056	0.126	0.175	0.221	0.319	0.357
	1		0.136	0.230	0.288	0.339	0.440	0.477
	2		0.217	0.324	0.386	0.438	0.537	0.573
	3		0.298	0.412	0.475	0.527	0.622	0.655
	4		0.379	0.497	0.559	0.609	0.698	0.728
	5		0.460	0.578	0.638	0.685	0.765	0.791
	6		0.540	0.656	0.712	0.755	0.825	0.848
	7		0.621	0.731	0.781	0.819	0.879	0.897
13	0		0.052	0.116	0.162	0.206	0.298	0.334
	1		0.126	0.213	0.268	0.316	0.413	0.449
	2		0.200	0.301	0.360	0.410	0.506	0.541
	3		0.275	0.384	0.444	0.495	0.588	0.621
	4		0.350	0.463	0.523	0.573	0.661	0.691
	5		0.425	0.539	0.598	0.645	0.727	0.755
	6		0.500	0.613	0.669	0.713	0.787	0.811
	7		0.575	0.684	0.736	0.776	0.841	0.862

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
14	0		0.048	0.109	0.152	0.193	0.280	0.315
	1		0.117	0.199	0.251	0.297	0.389	0.424
	2		0.186	0.281	0.337	0.385	0.478	0.512
	3		0.256	0.359	0.417	0.466	0.557	0.589
	4		0.326	0.434	0.492	0.540	0.627	0.658
	5		0.395	0.506	0.563	0.610	0.692	0.720
	6		0.465	0.575	0.631	0.675	0.751	0.777
	7		0.535	0.643	0.695	0.736	0.805	0.828
	8		0.605	0.708	0.757	0.794	0.854	0.873
15	0		0.045	0.102	0.142	0.181	0.264	0.298
	1		0.109	0.187	0.236	0.279	0.368	0.402
	2		0.174	0.264	0.317	0.363	0.453	0.486
	3		0.239	0.337	0.393	0.440	0.529	0.561
	4		0.305	0.407	0.464	0.511	0.597	0.627
	5		0.370	0.476	0.532	0.577	0.660	0.688
	6		0.435	0.542	0.596	0.640	0.718	0.744
	7		0.500	0.606	0.658	0.700	0.771	0.795
	8		0.565	0.668	0.718	0.756	0.821	0.841
	9		0.630	0.728	0.774	0.809	0.865	0.883
16	0		0.042	0.096	0.134	0.170	0.250	0.282
	1		0.103	0.176	0.222	0.264	0.349	0.381
	2		0.164	0.249	0.300	0.344	0.430	0.463
	3		0.225	0.318	0.371	0.417	0.503	0.534
	4		0.286	0.385	0.439	0.484	0.569	0.599
	5		0.347	0.449	0.504	0.548	0.630	0.658
	6		0.408	0.512	0.565	0.609	0.687	0.713
	7		0.469	0.573	0.625	0.667	0.739	0.764
	8		0.531	0.632	0.682	0.721	0.788	0.810
	9		0.592	0.690	0.737	0.773	0.834	0.853
17	0		0.040	0.090	0.127	0.162	0.237	0.268
	1		0.097	0.166	0.210	0.250	0.332	0.363
	2		0.154	0.235	0.284	0.326	0.410	0.441
	3		0.212	0.301	0.352	0.396	0.480	0.510
	4		0.269	0.364	0.416	0.461	0.543	0.573
	5		0.327	0.425	0.478	0.522	0.603	0.631
	6		0.385	0.485	0.537	0.580	0.658	0.685
	7		0.442	0.543	0.594	0.636	0.709	0.734
	8		0.500	0.600	0.650	0.689	0.758	0.781
	9		0.558	0.655	0.703	0.740	0.803	0.824
	10		0.615	0.709	0.754	0.788	0.845	0.863

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
18	0		0.038	0.086	0.120	0.153	0.226	0.255
	1		0.092	0.157	0.199	0.238	0.316	0.346
	2		0.146	0.223	0.269	0.310	0.391	0.422
	3		0.200	0.285	0.334	0.377	0.458	0.488
	4		0.255	0.345	0.396	0.439	0.520	0.549
	5		0.309	0.404	0.455	0.498	0.577	0.605
	6		0.364	0.460	0.512	0.554	0.631	0.658
	7		0.418	0.516	0.567	0.608	0.681	0.707
	8		0.473	0.570	0.620	0.659	0.729	0.753
	9		0.527	0.623	0.671	0.709	0.774	0.795
	10		0.582	0.675	0.721	0.756	0.816	0.835
19	0		0.036	0.081	0.114	0.146	0.215	0.243
	1		0.087	0.150	0.190	0.226	0.302	0.331
	2		0.138	0.212	0.257	0.296	0.374	0.404
	3		0.190	0.271	0.319	0.359	0.439	0.468
	4		0.242	0.329	0.378	0.419	0.498	0.527
	5		0.293	0.384	0.434	0.476	0.554	0.582
	6		0.345	0.438	0.489	0.530	0.606	0.633
	7		0.397	0.491	0.541	0.582	0.655	0.681
	8		0.448	0.543	0.592	0.632	0.702	0.726
	9		0.500	0.594	0.642	0.680	0.746	0.768
	10		0.551	0.644	0.690	0.726	0.788	0.808
	11		0.603	0.693	0.737	0.770	0.827	0.845
20	0		0.034	0.077	0.109	0.139	0.206	0.233
	1		0.083	0.142	0.181	0.216	0.289	0.317
	2		0.131	0.202	0.245	0.283	0.358	0.387
	3		0.181	0.259	0.304	0.344	0.421	0.449
	4		0.230	0.313	0.361	0.401	0.478	0.507
	5		0.279	0.366	0.415	0.456	0.532	0.560
	6		0.328	0.418	0.467	0.508	0.583	0.610
	7		0.377	0.469	0.518	0.558	0.631	0.657
	8		0.426	0.519	0.567	0.606	0.677	0.701
	9		0.475	0.568	0.615	0.653	0.720	0.743
	10		0.525	0.616	0.662	0.698	0.761	0.782
	11		0.574	0.663	0.707	0.741	0.800	0.819
	12		0.623	0.709	0.751	0.783	0.837	0.854
21	0		0.032	0.074	0.104	0.133	0.197	0.223
	1		0.079	0.136	0.173	0.207	0.277	0.304
	2		0.125	0.193	0.234	0.271	0.344	0.372
	3		0.172	0.247	0.291	0.329	0.404	0.432

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	4		0.219	0.299	0.345	0.384	0.460	0.488
	5		0.266	0.350	0.397	0.437	0.512	0.539
	6		0.313	0.400	0.448	0.487	0.561	0.588
	7		0.359	0.449	0.497	0.536	0.608	0.634
	8		0.406	0.497	0.544	0.583	0.653	0.677
	9		0.453	0.544	0.590	0.628	0.695	0.718
	10		0.500	0.590	0.636	0.672	0.736	0.758
	11		0.547	0.635	0.679	0.714	0.774	0.795
	12		0.594	0.680	0.722	0.755	0.810	0.829
22	0		0.031	0.071	0.099	0.127	0.189	0.214
	1		0.075	0.130	0.166	0.198	0.266	0.292
	2		0.120	0.185	0.224	0.259	0.331	0.358
	3		0.164	0.237	0.279	0.316	0.389	0.416
	4		0.209	0.287	0.331	0.369	0.443	0.470
	5		0.254	0.336	0.381	0.420	0.493	0.520
	6		0.299	0.383	0.430	0.468	0.541	0.567
	7		0.343	0.430	0.477	0.515	0.587	0.612
	8		0.388	0.476	0.523	0.561	0.630	0.655
	9		0.433	0.521	0.568	0.605	0.672	0.695
	10		0.478	0.566	0.611	0.647	0.712	0.734
	11		0.522	0.610	0.654	0.689	0.750	0.771
	12		0.567	0.653	0.695	0.729	0.786	0.805
	13		0.612	0.695	0.736	0.767	0.821	0.838
23	0		0.030	0.068	0.095	0.122	0.181	0.206
	1		0.072	0.125	0.159	0.190	0.256	0.281
	2		0.115	0.177	0.215	0.249	0.318	0.345
	3		0.157	0.227	0.268	0.304	0.374	0.401
	4		0.200	0.275	0.318	0.355	0.427	0.453
	5		0.243	0.322	0.366	0.404	0.476	0.502
	6		0.286	0.368	0.413	0.451	0.522	0.548
	7		0.329	0.413	0.459	0.496	0.567	0.592
	8		0.371	0.457	0.503	0.540	0.609	0.634
	9		0.414	0.501	0.546	0.583	0.650	0.674
	10		0.457	0.544	0.589	0.625	0.689	0.712
	11		0.500	0.586	0.630	0.665	0.727	0.748
	12		0.543	0.628	0.670	0.704	0.763	0.782
	13		0.586	0.669	0.710	0.742	0.797	0.815

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number cf failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
24	0		0.028	0.065	0.091	0.117	0.175	0.198
	1		0.069	0.120	0.153	0.183	0.246	0.271
	2		0.110	0.170	0.207	0.240	0.307	0.332
	3		0.151	0.218	0.258	0.292	0.361	0.387
	4		0.192	0.264	0.306	0.342	0.411	0.438
	5		0.233	0.309	0.352	0.389	0.459	0.485
	6		0.274	0.354	0.398	0.435	0.504	0.530
	7		0.315	0.397	0.412	0.479	0.548	0.573
	8		0.356	0.440	0.484	0.521	0.589	0.614
	9		0.397	0.482	0.526	0.563	0.629	0.653
	10		0.438	0.523	0.567	0.603	0.667	0.690
	11		0.479	0.564	0.608	0.642	0.704	0.726
	12		0.521	0.604	0.647	0.681	0.740	0.760
	13		0.562	0.644	0.685	0.718	01774	0.793
	14		0.603	0.683	0.723	0.754	0.806	0.824
25	0		0.027	0.062	0.088	0.113	0.168	0.191
	1		0.066	0.115	0.147	0.176	0.237	0.262
	2		0.106	0.163	0.199	0.231	0.296	0.321
	3		0.145	0.210	0.248	0.282	0.349	0.374
	4		0.184	0.254	0.295	0.330	0.398	0.424
	5		0.224	0.298	0.340	0.375	0.444	0.470
	6		0.263	0.340	0.383	0.420	0.488	0.514
	7		0.303	0.382	0.426	0.462	0.531	0.555
	8		0.342	0.423	0.467	0.504	0.571	0.595
	9		0.382	0.464	0.508	0.544	0.610	0.633
	10		0.421	0.504	0.548	0.583	0.648	0.670
	11		0.461	0.544	0.587	0.621	0.684	0.705
	12		0.500	0.583	0.625	0.659	0.719	0.739
	13		0.539	0.621	0.662	0.695	0.752	0.772
	14		0.579	0.659	0.699	0.730	0.784	0.803
	15		0.618	0.697	0.735	0.764	0.815	0.832
26	0		0.026	0.060	0.085	0.109	0.162	0.184
	1		0.064	0.111	0.142	0.170	0.229	0.253
	2		0.102	0.157	0.192	0.223	0.286	0.310
	3		0.139	0.202	0.239	0.272	0.337	0.362
	4		0.177	0.245	0.284	0.318	0.385	0.410
	5		0.215	0.287	0.328	0.363	0.430	0.455
	6		0.253	0.328	0.370	0.405	0.473	0.498
	7		0.291	0.369	0.411	0.447	0.514	0.538
	8		0.329	0.408	0.451	0.487	0.554	0.578
	9		0.267	0.448	0.491	0.526	0.592	0.615
	10		0.405	0.486	0.529	0.564	0.628	0.651

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.443	0.525	0.567	0.602	0.664	0.686
	12		0.481	0.562	0.604	0.638	0.698	0.719
	13		0.519	0.600	0.641	0.673	0.731	0.751
	14		0.557	0.637	0.676	0.708	0.763	0.782
	15		0.595	0.673	0.711	0.742	0.794	0.811
27	0		0.025	0.058	0.082	0.105	0.157	0.178
	1		0.061	0.107	0.137	0.164	0.222	0.245
	2		0.098	0.152	0.185	0.215	0.277	0.300
	3		0.134	0.195	0.231	0.263	0.326	0.351
	4		0.171	0.236	0.275	0.308	0.373	0.397
	5		0.207	0.277	0.317	0.351	0.417	0.441
	6		0.244	0.317	0.358	0.392	0.458	0.483
	7		0.281	0.356	0.397	0.432	0.498	0.523
	8		0.317	0.394	0.436	0.471	0.537	0.561
	9		0.354	0.432	0.475	0.509	0.574	0.597
	10		0.390	0.470	0.512	0.547	0.610	0.633
	11		0.427	0.507	0.549	0.583	0.645	0.667
	12		0.463	0.544	0.585	0.618	0.679	0.700
	13		0.500	0.580	0.620	0.653	0.711	0.731
	14		0.537	0.615	0.655	0.687	0.743	0.762
	15		0.573	0.651	0.689	0.720	0.773	0.791
	16		0.610	0.686	0.723	0.752	0.802	0.819
28	0		0.024	0.056	0.079	0.101	0.152	0.172
	1		0.059	0.103	0.132	0.159	0.215	0.237
	2		0.094	0.147	0.179	0.208	0.268	0.291
	3		0.130	0.188	0.223	0.254	0.316	0.340
	4		0.165	0.228	0.265	0.298	0.361	0.385
	5		0.200	0.268	0.306	0.339	0.404	0.428
	6		0.235	0.306	0.346	0.380	0.445	0.469
	7		0.271	0.344	0.385	0.419	0.484	0.508
	8		0.306	0.381	0.422	0.457	0.521	0.545
	9		0.341	0.418	0.459	0.494	0.558	0.581
	10		0.377	0.454	0.496	0.530	0.593	0.616
	11		0.412	0.490	0.532	0.565	0.627	0.649
	12		0.447	0.526	0.567	0.600	0.660	0.681
	13		0.482	0.561	0.601	0.634	0.692	0.713
	14		0.518	0.596	0.635	0.667	0.723	0.743
	15		0.553	0.630	0.669	0.699	0.753	0.772
	16		0.588	0.664	0.701	0.731	0.782	0.800

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
29	0		0.024	0.054	0.076	0.098	0.147	0.167
	1		0.057	0.100	0.128	0.153	0.208	0.230
	2		0.091	0.142	0.173	0.202	0.260	0.282
	3		0.125	0.182	0.216	0.246	0.307	0.330
	4		0.159	0.221	0.257	0.288	0.350	0.374
	5		0.193	0.259	0.297	0.329	0.392	0.416
	6		0.227	0.296	0.335	0.368	0.432	0.455
	7		0.261	0.333	0.372	0.406	0.470	0.493
	8		0.296	0.369	0.409	0.443	0.507	0.530
	9		0.330	0.405	0.445	0.479	0.542	0.565
	10		0.364	0.440	0.481	0.514	0.577	0.599
	11		0.398	0.475	0.515	0.549	0.610	0.632
	12		0.432	0.509	0.550	0.583	0.643	0.664
	13		0.466	0.543	0.583	0.616	0.674	0.695
	14		0.500	0.577	0.616	0.648	0.705	0.724
	15		0.534	0.610	0.649	0.680	0.705	0.724
	16		0.568	0.643	0.681	0.711	0.763	0.781
	17		0.602	0.676	0.712	0.741	0.791	0.808
30	0		0.023	0.052	0.074	0.095	0.142	0.162
	1		0.055	0.097	0.124	0.149	0.202	0.223
	2		0.088	0.137	0.168	0.195	0.252	0.274
	3		0.121	0.176	0.209	0.239	0.298	0.320
	4		0.154	0.214	0.249	0.280	0.340	0.363
	5		0.187	0.251	0.287	0.319	0.381	0.404
	6		0.220	0.287	0.324	0.357	0.420	0.443
	7		0.253	0.322	0.361	0.394	0.457	0.480
	8		0.286	0.358	0.397	0.430	0.493	0.516
	9		0.319	0.392	0.432	0.465	0.527	0.550
	10		0.352	0.426	0.466	0.499	0.561	0.583
	11		0.385	0.460	0.500	0.533	0.594	0.616
	12		0.418	0.494	0.533	0.566	0.626	0.647
	13		0.451	0.527	0.566	0.598	0.657	0.677
	14		0.484	0.559	0.599	0.630	0.687	0.707
	15		0.516	0.592	0.630	0.661	0.716	0.735
	16		0.549	0.624	0.662	0.695	0.745	0.763
	17		0.582	0.656	0.692	0.721	0.772	0.789
	18		0.615	0.687	0.723	0.750	0.799	0.815

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
31	0		0.022	0.051	0.072	0.092	0.138	0.157
	1		0.054	0.094	0.120	0.144	0.196	0.216
	2		0.085	0.133	0.163	0.189	0.245	0.266
	3		0.117	0.171	0.203	0.232	0.289	0.311
	4		0.149	0.207	0.242	0.271	0.330	0.353
	5		0.181	0.243	0.279	0.310	0.370	0.393
	6		0.213	0.278	0.315	0.347	0.408	0.431
	7		0.245	0.317	0.350	0.383	0.444	0.467
	8		0.277	0.347	0.385	0.418	0.479	0.502
	9		0.309	0.380	0.419	0.452	0.513	0.536
	10		0.340	0.413	0.453	0.485	0.546	0.569
	11		0.372	0.446	0.486	0.518	0.579	0.600
	12		0.404	0.479	0.518	0.550	0.610	0.631
	13		0.436	0.511	0.550	0.582	0.640	0.661
	14		0.468	0.543	0.582	0.613	0.610	0.690
	15		0.500	0.575	0.613	0.643	0.699	0.718
	16		0.532	0.606	0.643	0.673	0.727	0.745
	17		0.564	0.637	0.673	0.703	0.754	0.772
	18		0.596	0.667	0.703	0.731	0.780	0.797
32	0		0.021	0.049	0.069	0.089	0.134	0.153
	1		0.052	0.091	0.116	0.139	0.190	0.210
	2		0.083	0.129	0.158	0.183	0.238	0.259
	3		0.114	0.166	0.197	0.224	0.281	0.303
	4		0.144	0.201	0.234	0.263	0.322	0.344
	5		0.175	0.236	0.271	0.300	0.360	0.383
	6		0.206	0.270	0.306	0.336	0.397	0.419
	7		0.237	0.303	0.340	0.371	0.433	0.455
	8		0.268	0.336	0.374	0.406	0.467	0.489
	9		0.299	0.369	0.407	0.439	0.500	0.522
	10		0.330	0.401	0.440	0.472	0.532	0.554
	11		0.361	0.433	0.472	0.504	0.564	0.585
	12		0.392	0.465	0.504	0.535	0.595	0.616
	13		0.423	0.496	0.535	0.566	0.624	0.645
	14		0.454	0.527	0.566	0.596	0.654	0.674
	15		0.485	0.558	0.596	0.626	0.682	0.701
	16		0.515	0.589	0.626	0.655	0.710	0.728
	17		0.546	0.619	0.655	0.684	0.736	0.754
	18		0.577	0.649	0.684	0.712	0.763	0.780
	19		0.608	0.678	0.713	0.740	0.788	0.804

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
33	0		0.021	0.048	0.067	0.087	0.130	0.148
	1		0.050	0.088	0.112	0.136	0.185	0.204
	2		0.080	0.125	0.153	0.179	0.231	0.252
	3		0.110	0.161	0.191	0.219	0.273	0.295
	4		0.140	0.195	0.228	0.256	0.313	0.335
	5		0.170	0.229	0.263	0.293	0.351	0.373
	6		0.200	0.262	0.297	0.328	0.387	0.409
	7		0.230	0.295	0.331	0.362	0.421	0.443
	8		0.260	0.327	0.364	0.395	0.455	0.477
	9		0.290	0.359	0.396	0.428	0.487	0.509
	10		0.320	0.390	0.428	0.460	0.519	0.541
	11		0.350	0.421	0.459	0.491	0.550	0.571
	12		0.380	0.452	0.490	0.522	0.580	0.601
	13		0.410	0.482	0.521	0.552	0.609	0.630
	14		0.440	0.513	0.551	0.581	0.638	0.658
	15		0.470	0.543	0.580	0.611	0.666	0.685
	16		0.500	0.572	0.610	0.639	0.693	0.712
	17		0.530	0.602	0.638	0.667	0.720	0.738
	18		0.560	0.631	0.666	0.695	0.746	0.763
	19		0.590	0.660	0.695	0.722	0.771	0.787
34	0		0.020	0.046	0.065	0.084	0.127	0.144
	1		0.049	0.086	0.110	0.132	0.180	0.199
	2		0.078	0.122	0.149	0.174	0.225	0.245
	3		0.107	0.156	0.186	0.213	0.266	0.287
	4		0.136	0.190	0.221	0.249	0.305	0.326
	5		0.165	0.223	0.256	0.285	0.342	0.363
	6		0.194	0.255	0.289	0.319	0.377	0.398
	7		0.223	0.286	0.322	0.352	0.411	0.432
	8		0.252	0.318	0.354	0.385	0.443	0.465
	9		0.282	0.349	0.385	0.416	0.475	0.497
	10		0.311	0.379	0.416	0.448	0.506	0.528
	11		0.340	0.410	0.447	0.478	0.537	0.558
	12		0.369	0.440	0.477	0.508	0.566	0.587
	13		0.398	0.469	0.507	0.538	0.595	0.615
	14		0.427	0.499	0.536	0.567	0.623	0.643
	15		0.456	0.528	0.565	0.595	0.650	0.670
	16		0.485	0.557	0.594	0.623	0.677	0.696
	17		0.515	0.586	0.622	0.651	0.704	0.722
	18		0.544	0.614	0.650	0.678	0.729	0.747
	19		0.573	0.642	0.677	0.705	0.754	0.771
	20		0.602	0.670	0.704	0.731	0.778	0.794

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
35	0		0.020	0.045	0.064	0.082	0.123	0.140
	1		0.047	0.083	0.107	0.129	0.175	0.194
	2		0.076	0.118	0.145	0.169	0.219	0.239
	3		0.104	0.152	0.181	0.207	0.259	0.280
	4		0.132	0.185	0.216	0.243	0.297	0.318
	5		0.160	0.217	0.249	0.277	0.333	0.354
	6		0.189	0.248	0.282	0.311	0.367	0.389
	7		0.217	0.279	0.313	0.343	0.400	0.422
	8		0.245	0.309	0.345	0.375	0.433	0.454
	9		0.274	0.339	0.375	0.406	0.464	0.485
	10		0.302	0.369	0.406	0.436	0.494	0.515
	11		0.330	0.399	0.435	0.466	0.524	0.545
	12		0.359	0.428	0.465	0.496	0.553	0.574
	13		0.387	0.457	0.494	0.524	0.581	0.601
	14		0.415	0.485	0.522	0.553	0.609	0.629
	15		0.443	0.514	0.551	0.581	0.636	0.655
	16		0.472	0.542	0.579	0.608	0.662	0.681
	17		0.500	0.570	0.606	0.635	0.688	0.706
	18		0.528	0.598	0.634	0.662	0.713	0.731
	19		0.557	0.626	0.660	0.688	0.738	0.755
	20		0.585	0.653	0.687	0.714	0.762	0.778
	21		0.613	0.680	0.713	0.740	0.785	0.801
36	0		0.019	0.044	0.062	0.080	0.120	0.137
	1		0.046	0.081	0.104	0.125	0.171	0.189
	2		0.074	0.115	0.141	0.165	0.214	0.233
	3		0.101	0.148	0.176	0.202	0.253	0.273
	4		0.129	0.180	0.210	0.236	0.290	0.310
	5		0.156	0.211	0.242	0.270	0.325	0.346
	6		0.184	0.241	0.274	0.303	0.358	0.379
	7		0.211	0.271	0.305	0.334	0.391	0.412
	8		0.239	0.301	0.336	0.365	0.422	0.443
	9		0.266	0.330	0.366	0.396	0.453	0.474
	10		0.294	0.359	0.395	0.425	0.483	0.504
	11		0.321	0.388	0.424	0.455	0.512	0.533
	12		0.349	0.417	0.453	0.483	0.540	0.561
	13		0.376	0.445	0.481	0.512	0.568	0.588
	14		0.404	0.473	0.509	0.540	0.595	0.615
	15		0.431	0.501	0.537	0.567	0.622	0.641
	16		0.459	0.528	0.564	0.594	0.648	0.667
	17		0.486	0.556	0.591	0.620	0.673	0.692
	18		0.514	0.583	0.618	0.647	0.698	0.716
	19		0.541	0.610	0.645	0.672	0.722	0.740
	20		0.569	0.637	0.671	0.698	0.746	0.763
	21		0.596	0.663	0.696	0.723	0.769	0.785

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
37	0		0.019	0.043	0.060	0.078	0.117	0.133
	1		0.045	0.079	0.101	0.122	0.166	0.184
	2		0.072	0.112	0.138	0.161	0.208	0.227
	3		0.098	0.144	0.172	0.196	0.247	0.266
	4		0.125	0.175	0.205	0.231	0.283	0.303
	5		0.152	0.205	0.236	0.263	0.317	0.337
	6		0.179	0.235	0.267	0.295	0.350	0.371
	7		0.205	0.264	0.298	0.326	0.382	0.402
	8		0.232	0.293	0.327	0.356	0.412	0.433
	9		0.259	0.322	0.357	0.386	0.442	0.463
	10		0.286	0.350	0.385	0.415	0.472	0.492
	11		0.313	0.378	0.414	0.444	0.500	0.521
	12		0.339	0.406	0.442	0.472	0.555	0.575
	13		0.366	0.434	0.470	0.500	0.555	0.575
	14		0.393	0.461	0.497	0.527	0.582	0.602
	15		0.420	0.488	0.524	0.554	0.608	0.627
	16		0.446	0.515	0.551	0.580	0.634	0.653
	17		0.473	0.542	0.577	0.606	0.659	0.677
	18		0.500	0.568	0.604	0.632	0.683	0.701
	19		0.527	0.595	0.629	0.657	0.707	0.725
	20		0.554	0.621	0.655	0.682	0.731	0.748
	21		0.580	0.647	0.680	0.707	0.754	0.770
	22		0.607	0.673	0.705	0.730	0.776	0.792
38	0		0.018	0.041	0.059	0.076	0.114	0.130
	1		0.044	0.077	0.099	0.119	0.162	0.180
	2		0.070	0.109	0.134	0.157	0.203	0.222
	3		0.096	0.140	0.167	0.192	0.241	0.260
	4		0.122	0.171	0.199	0.225	0.276	0.296
	5		0.148	0.200	0.230	0.257	0.310	0.330
	6		0.174	0.229	0.261	0.288	0.342	0.362
	7		0.200	0.258	0.290	0.318	0.373	0.393
	8		0.226	0.286	0.319	0.348	0.403	0.424
	9		0.252	0.314	0.348	0.377	0.432	0.453
	10		0.278	0.342	0.376	0.405	0.461	0.482
	11		0.304	0.369	0.404	0.433	0.489	0.509
	12		0.330	0.396	0.431	0.461	0.516	0.537
	13		0.357	0.423	0.458	0.488	0.543	0.563
	14		0.383	0.450	0.485	0.515	0.569	0.589
	15		0.409	0.476	0.512	0.541	0.595	0.614
	16		0.435	0.503	0.538	0.567	0.620	0.639
	17		0.461	0.529	0.564	0.593	0.645	0.663
	18		0.487	0.556	0.589	0.618	0.669	0.687

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	19		0.513	0.580	0.615	0.643	0.693	0.710
	20		0.539	0.606	0.640	0.667	0.716	0.733
	21		0.565	0.631	0.665	0.691	0.739	0.755
	22		0.591	0.657	0.689	0.715	0.761	0.777
39	0		0.018	0.040	0.057	0.074	0.111	0.127
	1		0.043	0.075	0.096	0.116	0.158	0.176
	2		0.068	0.107	0.131	0.153	0.199	0.217
	3		0.093	0.137	0.163	0.187	0.235	0.254
	4		0.119	0.166	0.195	0.220	0.270	0.289
	5		0.144	0.195	0.225	0.251	0.303	0.322
	6		0.170	0.223	0.254	0.281	0.334	0.354
	7		0.195	0.251	0.283	0.311	0.364	0.385
	8		0.220	0.279	0.312	0.340	0.394	0.414
	9		0.246	0.306	0.340	0.368	0.423	0.443
	10		0.271	0.333	0.367	0.396	0.451	0.471
	11		0.297	0.360	0.394	0.423	0.478	0.499
	12		0.322	0.386	0.421	0.450	0.505	0.525
	13		0.347	0.413	0.448	0.477	0.531	0.551
	14		0.373	0.439	0.474	0.503	0.557	0.577
	15		0.398	0.465	0.500	0.529	0.583	0.602
	16		0.424	0.491	0.526	0.554	0.607	0.626
	17		0.449	0.516	0.551	0.579	0.632	0.650
	18		0.475	0.541	0.576	0.604	0.655	0.674
	19		0.500	0.567	0.601	0.629	0.679	0.697
	20		0.525	0.592	0.625	0.653	0.702	0.719
	21		0.551	0.617	0.650	0.677	0.724	0.741
	22		0.576	0.641	0.674	0.700	0.746	0.762
	23		0.602	0.666	0.698	0.723	0.768	0.783
40	0		0.017	0.039	0.056	0.072	0.109	0.124
	1		0.042	0.073	0.094	0.113	0.155	0.172
	2		0.066	0.104	0.128	0.149	0.194	0.212
	3		0.091	0.134	0.159	0.183	0.230	0.248
	4		0.116	0.162	0.190	0.214	0.264	0.283
	5		0.141	0.190	0.220	0.245	0.296	0.315
	6		0.165	0.218	0.248	0.275	0.327	0.346
	7		0.190	0.245	0.277	0.304	0.356	0.376
	8		0.215	0.272	0.305	0.332	0.385	0.405
	9		0.240	0.299	0.332	0.360	0.414	0.434
	10		0.265	0.325	0.359	0.387	0.441	0.461

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.289	0.351	0.385	0.414	0.468	0.488
	12		0.314	0.377	0.412	0.440	0.494	0.514
	13		0.339	0.403	0.438	0.466	0.520	0.540
	14		0.364	0.429	0.463	0.492	0.546	0.565
	15		0.388	0.454	0.489	0.517	0.570	0.590
	16		0.413	0.479	0.514	0.542	0.595	0.614
	17		0.438	0.504	0.539	0.567	0.619	0.637
	18		0.463	0.529	0.563	0.591	0.642	0.661
	19		0.488	0.554	0.588	0.615	0.665	0.683
	20		0.512	0.578	0.612	0.639	0.688	0.705
	21		0.537	0.602	0.636	0.662	0.710	0.727
	22		0.562	0.627	0.659	0.685	0.732	0.748
	23		0.587	0.651	0.683	0.708	0.754	0.769
	24		0.612	0.674	0.706	0.731	0.774	0.790
41	0		0.017	0.038	0.055	0.070	0.106	0.121
	1		0.041	0.071	0.092	0.111	0.151	0.168
	2		0.065	0.101	0.125	0.146	0.190	0.207
	3		0.089	0.130	0.156	0.178	0.225	0.243
	4		0.113	0.159	0.186	0.210	0.258	0.276
	5		0.137	0.186	0.215	0.239	0.289	0.308
	6		0.161	0.213	0.243	0.269	0.320	0.339
	7		0.186	0.240	0.270	0.297	0.349	0.368
	8		0.210	0.266	0.298	0.325	0.377	0.397
	9		0.234	0.292	0.324	0.352	0.405	0.424
	10		0.258	0.318	0.351	0.379	0.432	0.452
	11		0.282	0.343	0.377	0.405	0.458	0.478
	12		0.306	0.369	0.402	0.431	0.484	0.504
	13		0.331	0.394	0.428	0.456	0.510	0.529
	14		0.355	0.419	0.453	0.481	0.534	0.554
	15		0.379	0.444	0.478	0.506	0.559	0.578
	16		0.403	0.468	0.502	0.531	0.583	0.602
	17		0.427	0.493	0.527	0.555	0.607	0.625
	18		0.452	0.517	0.551	0.579	0.630	0.648
	19		0.476	0.541	0.575	0.602	0.652	0.670
	20		0.500	0.565	0.598	0.626	0.675	0.692
	21		0.524	0.589	0.622	0.649	0.697	0.714
	22		0.549	0.613	0.645	0.671	0.718	0.735
	23		0.573	0.636	0.668	0.694	0.740	0.755
	24		0.597	0.659	0.691	0.716	0.760	0.776

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
42	0		0.016	0.038	0.053	0.068	0.104	0.119
	1		0.040	0.070	0.089	0.108	0.148	0.164
	2		0.063	0.099	0.122	0.142	0.185	0.203
	3		0.087	0.127	0.152	0.174	0.220	0.238
	4		0.110	0.155	0.181	0.204	0.252	0.271
	5		0.134	0.182	0.210	0.234	0.283	0.302
	6		0.158	0.208	0.237	0.262	0.313	0.332
	7		0.181	0.234	0.264	0.290	0.341	0.361
	8		0.205	0.260	0.291	0.317	0.369	0.389
	9		0.228	0.285	0.317	0.344	0.396	0.416
	10		0.252	0.311	0.343	0.370	0.423	0.443
	11		0.276	0.336	0.368	0.396	0.449	0.468
	12		0.299	0.360	0.394	0.421	0.474	0.494
	13		0.323	0.385	0.418	0.446	0.499	0.519
	14		0.346	0.409	0.443	0.471	0.524	0.543
	15		0.370	0.434	0.467	0.495	0.548	0.567
	16		0.394	0.458	0.492	0.519	0.571	0.590
	17		0.417	0.482	0.515	0.543	0.595	0.613
	18		0.441	0.505	0.539	0.566	0.617	0.636
	19		0.465	0.529	0.563	0.589	0.640	0.658
	20		0.488	0.553	0.586	0.612	0.662	0.679
	21		0.519	0.576	0.609	0.635	0.684	0.701
	22		0.535	0.599	0.632	0.657	0.705	0.722
	23		0.559	0.622	0.654	0.679	0.726	0.742
	24		0.583	0.645	0.677	0.701	0.746	0.762
	25		0.606	0.668	0.699	0.723	0.767	0.782
43	0		0.016	0.037	0.052	0.067	0.102	0.116
	1		0.039	0.068	0.087	0.106	0.145	0.160
	2		0.062	0.097	0.119	0.139	0.181	0.198
	3		0.085	0.125	0.149	0.171	0.215	0.233
	4		0.108	0.151	0.177	0.200	0.247	0.265
	5		0.131	0.178	0.205	0.229	0.277	0.291
	6		0.154	0.203	0.232	0.257	0.306	0.325
	7		0.177	0.229	0.259	0.284	0.334	0.353
	8		0.200	0.254	0.285	0.311	0.362	0.381
	9		0.223	0.279	0.310	0.337	0.388	0.408
	10		0.246	0.304	0.335	0.362	0.414	0.434
	11		0.269	0.328	0.360	0.388	0.440	0.459
	12		0.292	0.352	0.385	0.413	0.465	0.484
	13		0.315	0.376	0.409	0.437	0.489	0.509
	14		0.338	0.400	0.434	0.461	0.513	0.532
	15		0.362	0.424	0.458	0.485	0.537	0.556

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	16		0.385	0.448	0.481	0.509	0.560	0.579
	17		0.408	0.471	0.505	0.532	0.583	0.602
	18		0.431	0.494	0.528	0.555	0.606	0.624
	19		0.454	0.518	0.551	0.578	0.628	0.646
	20		0.477	0.541	0.574	0.601	0.650	0.667
	21		0.500	0.564	0.596	0.623	0.671	0.688
	22		0.523	0.586	0.619	0.645	0.692	0.709
	23		0.546	0.609	0.641	0.667	0.713	0.729
	24		0.569	0.631	0.663	0.688	0.733	0.749
	25		0.592	0.654	0.685	0.709	0.753	0.768
44	0		0.016	0.036	0.051	0.066	0.099	0.113
	1		0.038	0.067	0.086	0.103	0.142	0.157
	2		0.060	0.095	0.116	0.136	0.178	0.194
	3		0.083	0.122	0.146	0.167	0.211	0.228
	4		0.105	0.148	0.174	0.196	0.242	0.259
	5		0.128	0.174	0.201	0.224	0.271	0.290
	6		0.150	0.199	0.227	0.252	0.300	0.318
	7		0.173	0.224	0.253	0.278	0.328	0.346
	8		0.196	0.249	0.279	0.304	0.354	0.373
	9		0.218	0.273	0.304	0.330	0.381	0.400
	10		0.241	0.297	0.328	0.355	0.406	0.425
	11		0.263	0.321	0.353	0.380	0.431	0.450
	12		0.286	0.345	0.377	0.404	0.456	0.475
	13		0.308	0.368	0.401	0.428	0.480	0.499
	14		0.331	0.392	0.425	0.452	0.504	0.522
	15		0.353	0.415	0.448	0.475	0.527	0.546
	16		0.376	0.438	0.471	0.499	0.550	0.568
	17		0.399	0.461	0.494	0.521	0.572	0.590
	18		0.421	0.484	0.517	0.544	0.594	0.612
	19		0.444	0.507	0.540	0.567	0.616	0.634
	20		0.466	0.529	0.562	0.589	0.638	0.655
	21		0.489	0.552	0.584	0.611	0.659	0.676
	22		0.511	0.574	0.606	0.632	0.680	0.696
	23		0.534	0.596	0.628	0.654	0.700	0.716
	24		0.556	0.618	0.650	0.675	0.720	0.736
	25		0.579	0.640	0.671	0.696	0.740	0.755
	26		0.601	0.662	0.692	0.716	0.759	0.774

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
45	0		0.015	0.035	0.050	0.064	0.097	0.111
	1		0.037	0.065	0.084	0.101	0.139	0.154
	2		0.059	0.093	0.114	0.133	0.174	0.190
	3		0.081	0.119	0.142	0.163	0.206	0.223
	4		0.103	0.145	0.170	0.192	0.237	0.254
	5		0.125	0.170	0.196	0.220	0.266	0.284
	6		0.147	0.195	0.222	0.246	0.294	0.312
	7		0.169	0.219	0.248	0.272	0.321	0.339
	8		0.191	0.243	0.273	0.298	0.347	0.366
	9		0.213	0.267	0.297	0.323	0.373	0.392
	10		0.235	0.291	0.322	0.348	0.398	0.417
	11		0.257	0.314	0.346	0.372	0.423	0.442
	12		0.279	0.337	0.369	0.396	0.447	0.466
	13		0.301	0.361	0.393	0.420	0.471	0.489
	14		0.324	0.384	0.416	0.443	0.494	0.513
	15		0.346	0.406	0.439	0.466	0.517	0.535
	16		0.368	0.429	0.462	0.489	0.539	0.558
	17		0.390	0.451	0.484	0.511	0.562	0.580
	18		0.412	0.474	0.507	0.533	0.583	0.601
	19		0.434	0.496	0.529	0.555	0.605	0.623
	20		0.456	0.518	0.551	0.577	0.626	0.643
	21		0.478	0.540	0.573	0.599	0.647	0.664
	22		0.500	0.562	0.594	0.620	0.667	0.684
	23		0.522	0.584	0.616	0.641	0.688	0.704
	24		0.544	0.606	0.637	0.662	0.708	0.723
	25		0.566	0.627	0.658	0.683	0.727	0.743
	26		0.588	0.648	0.679	0.703	0.746	0.761
	27		0.610	0.670	0.699	0.723	0.765	0.780
46	0		0.015	0.034	0.049	0.063	0.095	0.109
	1		0.036	0.064	0.082	0.099	0.136	0.151
	2		0.058	0.091	0.112	0.131	0.170	0.186
	3		0.079	0.117	0.140	0.160	0.202	0.219
	4		0.101	0.142	0.166	0.188	0.232	0.249
	5		0.122	0.166	0.192	0.215	0.261	0.278
	6		0.144	0.191	0.218	0.241	0.288	0.306
	7		0.166	0.215	0.243	0.267	0.315	0.333
	8		0.187	0.238	0.267	0.292	0.341	0.359
	9		0.209	0.262	0.291	0.317	0.366	0.384
	10		0.230	0.285	0.315	0.341	0.391	0.409
	11		0.252	0.308	0.339	0.365	0.415	0.433
	12		0.273	0.330	0.362	0.388	0.439	0.457
	13		0.295	0.353	0.385	0.411	0.462	0.480

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	14		0.317	0.376	0.408	0.434	0.485	0.503
	15		0.338	0.398	0.430	0.457	0.507	0.526
	16		0.360	0.420	0.452	0.479	0.529	0.548
	17		0.381	0.442	0.475	0.501	0.551	0.569
	18		0.403	0.464	0.497	0.523	0.573	0.591
	19		0.424	0.486	0.518	0.545	0.594	0.612
	20		0.446	0.508	0.540	0.566	0.615	0.632
	21		0.468	0.529	0.561	0.587	0.635	0.653
	22		0.489	0.551	0.583	0.608	0.656	0.672
	23		0.511	0.572	0.604	0.629	0.676	0.692
	24		0.532	0.593	0.625	0.650	0.695	0.711
	25		0.554	0.615	0.645	0.670	0.715	0.730
	26		0.576	0.636	0.666	0.690	0.734	0.749
	27		0.597	0.656	0.686	0.710	0.752	0.767
47	0		0.015	0.034	0.048	0.062	0.093	0.107
	1		0.035	0.062	0.080	0.097	0.133	0.148
	2		0.056	0.089	0.109	0.128	0.167	0.183
	3		0.078	0.114	0.137	0.157	0.198	0.215
	4		0.099	0.139	0.163	0.184	0.228	0.244
	5		0.120	0.163	0.188	0.211	0.256	0.273
	6		0.141	0.187	0.213	0.237	0.283	0.300
	7		0.162	0.210	0.238	0.262	0.309	0.327
	8		0.183	0.233	0.262	0.286	0.334	0.352
	9		0.204	0.256	0.285	0.310	0.359	0.377
	10		0.225	0.279	0.309	0.334	0.383	0.402
	11		0.247	0.301	0.332	0.358	0.407	0.425
	12		0.268	0.324	0.355	0.381	0.430	0.449
	13		0.289	0.346	0.377	0.403	0.453	0.472
	14		0.310	0.368	0.399	0.426	0.476	0.494
	15		0.331	0.390	0.422	0.448	0.498	0.516
	16		0.352	0.412	0.444	0.470	0.520	0.538
	17		0.373	0.433	0.465	0.492	0.541	0.559
	18		0.394	0.455	0.487	0.513	0.563	0.580
	19		0.416	0.476	0.508	0.535	0.583	0.601
	20		0.437	0.498	0.530	0.556	0.604	0.621
	21		0.458	0.519	0.551	0.577	0.624	0.641
	22		0.479	0.540	0.571	0.597	0.644	0.661
	23		0.500	0.561	0.592	0.618	0.664	0.681
	24		0.521	0.582	0.613	0.638	0.683	0.700
	25		0.542	0.602	0.633	0.658	0.703	0.718
	26		0.563	0.623	0.653	0.678	0.721	0.737
	27		0.585	0.644	0.673	0.697	0.740	0.755
	28		0.606	0.664	0.693	0.717	0.758	0.773

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
48	0		0.014	0.033	0.047	0.061	0.091	0.105
	1		0.035	0.061	0.079	0.095	0.130	0.145
	2		0.055	0.087	0.107	0.125	0.164	0.179
	3		0.076	0.112	0.134	0.154	0.194	0.210
	4		0.097	0.136	0.160	0.181	0.223	0.240
	5		0.117	0.160	0.185	0.207	0.251	0.268
	6		0.138	0.183	0.209	0.232	0.277	0.295
	7		0.159	0.206	0.233	0.257	0.303	0.321
	8		0.179	0.229	0.257	0.281	0.328	0.346
	9		0.200	0.251	0.280	0.304	0.352	0.370
	10		0.221	0.273	0.303	0.328	0.376	0.394
	11		0.241	0.295	0.325	0.351	0.400	0.418
	12		0.262	0.317	0.348	0.373	0.423	0.441
	13		0.283	0.339	0.370	0.396	0.445	0.463
	14		0.303	0.361	0.392	0.418	0.467	0.485
	15		0.324	0.382	0.414	0.440	0.489	0.507
	16		0.345	0.404	0.435	0.461	0.511	0.529
	17		0.366	0.425	0.456	0.483	0.532	0.550
	18		0.386	0.446	0.478	0.504	0.553	0.570
	19		0.407	0.467	0.499	0.525	0.573	0.591
	20		0.428	0.488	0.519	0.545	0.594	0.611
	21		0.448	0.509	0.540	0.566	0.614	0.631
	22		0.469	0.529	0.561	0.586	0.633	0.650
	23		0.490	0.550	0.581	0.606	0.653	0.669
	24		0.510	0.570	0.601	0.626	0.672	0.688
	25		0.531	0.591	0.621	0.646	0.691	0.707
	26		0.552	0.611	0.641	0.666	0.710	0.725
	27		0.572	0.631	0.661	0.685	0.728	0.743
	28		0.593	0.651	0.681	0.704	0.746	0.761
49	0		0.014	0.032	0.046	0.059	0.090	0.102
	1		0.034	0.060	0.077	0.093	0.128	0.142
	2		0.054	0.085	0.105	0.123	0.161	0.176
	3		0.074	0.110	0.131	0.151	0.191	0.207
	4		0.095	0.133	0.157	0.177	0.219	0.235
	5		0.115	0.157	0.181	0.203	0.246	0.263
	6		0.135	0.179	0.205	0.227	0.272	0.289
	7		0.155	0.202	0.229	0.252	0.297	0.315
	8		0.176	0.224	0.252	0.275	0.322	0.340
	9		0.196	0.246	0.274	0.299	0.346	0.364
	10		0.216	0.268	0.297	0.322	0.369	0.387

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.237	0.290	0.319	0.344	0.392	0.410
	12		0.257	0.311	0.341	0.366	0.415	0.433
	13		0.277	0.333	0.363	0.388	0.437	0.455
	14		0.297	0.354	0.384	0.410	0.459	0.477
	15		0.318	0.375	0.406	0.432	0.481	0.498
	16		0.338	0.396	0.427	0.453	0.502	0.520
	17		0.358	0.417	0.448	0.474	0.523	0.540
	18		0.378	0.437	0.469	0.495	0.543	0.561
	19		0.399	0.458	0.489	0.515	0.563	0.581
	20		0.419	0.479	0.510	0.536	0.583	0.601
	21		0.439	0.499	0.530	0.556	0.603	0.620
	22		0.459	0.519	0.550	0.576	0.623	0.639
	23		0.480	0.539	0.570	0.596	0.642	0.658
	24		0.500	0.560	0.590	0.615	0.661	0.677
	25		0.520	0.580	0.610	0.635	0.680	0.695
	26		0.541	0.600	0.630	0.654	0.698	0.714
	27		0.561	0.619	0.649	0.673	0.716	0.731
	28		0.581	0.639	0.668	0.692	0.734	0.749
	29		0.601	0.659	0.688	0.711	0.752	0.766
50	0		0.014	0.032	0.045	0.058	0.088	0.101
	1		0.033	0.059	0.076	0.091	0.126	0.139
	2		0.053	0.084	0.103	0.121	0.158	0.173
	3		0.073	0.108	0.129	0.148	0.187	0.203
	4		0.093	0.131	0.154	0.174	0.215	0.231
	5		0.113	0.154	0.178	0.199	0.242	0.258
	6		0.133	0.176	0.201	0.223	0.267	0.284
	7		0.152	0.198	0.224	0.247	0.292	0.309
	8		0.172	0.220	0.247	0.270	0.316	0.333
	9		0.192	0.241	0.269	0.293	0.340	0.357
	10		0.212	0.263	0.291	0.316	0.363	0.380
	11		0.232	0.284	0.313	0.338	0.385	0.403
	12		0.252	0.305	0.335	0.360	0.408	0.425
	13		0.272	0.326	0.356	0.381	0.430	0.447
	14		0.291	0.347	0.377	0.403	0.451	0.469
	15		0.311	0.368	0.398	0.424	0.472	0.490
	16		0.331	0.388	0.419	0.445	0.493	0.511
	17		0.351	0.409	0.440	0.465	0.514	0.531
	18		0.371	0.429	0.460	0.486	0.534	0.551
	19		0.391	0.449	0.480	0.506	0.554	0.571
	20		0.411	0.469	0.501	0.526	0.574	0.591

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	21		0.430	0.490	0.521	0.546	0.593	0.610
	22		0.450	0.510	0.540	0.566	0.612	0.629
	23		0.470	0.529	0.560	0.585	0.631	0.648
	24		0.490	0.549	0.580	0.605	0.650	0.666
	25		0.510	0.561	0.599	0.624	0.669	0.684
	26		0.530	0.588	0.618	0.643	0.687	0.702
	27		0.550	0.608	0.638	0.662	0.705	0.720
	28		0.570	0.627	0.657	0.680	0.723	0.737
	29		0.589	0.647	0.675	0.699	0.740	0.755
	30		0.609	0.666	0.694	0.717	0.757	0.771
55	0		0.013	0.029	0.041	0.053	0.080	0.092
	1		0.030	0.053	0.069	0.083	0.015	0.127
	2		0.048	0.076	0.094	0.110	0.144	0.158
	3		0.066	0.098	0.117	0.135	0.171	0.186
	4		0.084	0.119	0.140	0.159	0.197	0.212
	5		0.102	0.140	0.162	0.182	0.221	0.237
	6		0.121	0.160	0.184	0.204	0.245	0.261
	7		0.139	0.181	0.205	0.226	0.268	0.284
	8		0.157	0.200	0.226	0.247	0.290	0.306
	9		0.175	0.220	0.246	0.268	0.312	0.328
	10		0.193	0.240	0.266	0.289	0.333	0.350
	11		0.211	0.259	0.286	0.309	0.354	0.371
	12		0.229	0.279	0.306	0.329	0.375	0.391
	13		0.247	0.298	0.326	0.349	0.395	0.412
	14		0.265	0.317	0.345	0.369	0.415	0.432
	15		0.283	0.336	0.364	0.388	0.434	0.451
	16		0.301	0.355	0.383	0.408	0.454	0.471
	17		0.319	0.373	0.402	0.427	0.473	0.490
	18		0.337	0.392	0.421	0.446	0.492	0.509
	19		0.355	0.411	0.440	0.465	0.511	0.527
	20		0.373	0.429	0.459	0.483	0.529	0.546
	21		0.391	0.447	0.477	0.502	0.547	0.564
	22		0.409	0.466	0.495	0.520	0.565	0.582
	23		0.428	0.484	0.514	0.538	0.583	0.599
	24		0.446	0.502	0.532	0.556	0.601	0.617
	25		0.464	0.520	0.550	0.574	0.618	0.634

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	21		0.430	0.490	0.521	0.546	0.593	0.610
	22		0.450	0.510	0.540	0.566	0.612	0.629
	23		0.470	0.529	0.560	0.585	0.631	0.648
	24		0.490	0.549	0.580	0.605	0.650	0.666
	25		0.510	0.561	0.599	0.624	0.669	0.684
	26		0.530	0.586	0.618	0.643	0.687	0.702
	27		0.550	0.608	0.638	0.662	0.705	0.720
	28		0.570	0.627	0.657	0.680	0.723	0.737
	29		0.589	0.647	0.675	0.699	0.740	0.755
	30		0.609	0.666	0.694	0.717	0.757	0.771
55	0		0.013	0.029	0.041	0.053	0.080	0.092
	1		0.030	0.053	0.069	0.083	0.015	0.127
	2		0.048	0.076	0.094	0.110	0.144	0.158
	3		0.066	0.098	0.117	0.135	0.171	0.186
	4		0.084	0.119	0.140	0.159	0.197	0.212
	5		0.102	0.140	0.162	0.182	0.221	0.237
	6		0.121	0.160	0.184	0.204	0.245	0.261
	7		0.139	0.181	0.205	0.226	0.268	0.284
	8		0.157	0.200	0.226	0.247	0.290	0.306
	9		0.175	0.220	0.246	0.268	0.312	0.328
	10		0.193	0.240	0.266	0.289	0.333	0.350
	11		0.211	0.259	0.286	0.309	0.354	0.371
	12		0.229	0.279	0.306	0.329	0.375	0.391
	13		0.247	0.298	0.326	0.349	0.395	0.412
	14		0.265	0.317	0.345	0.369	0.415	0.432
	15		0.283	0.336	0.364	0.388	0.434	0.451
	16		0.301	0.355	0.383	0.408	0.454	0.471
	17		0.319	0.373	0.402	0.427	0.473	0.490
	18		0.337	0.392	0.421	0.446	0.492	0.509
	19		0.355	0.411	0.440	0.465	0.511	0.527
	20		0.373	0.429	0.459	0.483	0.529	0.546
	21		0.391	0.447	0.477	0.502	0.547	0.564
	22		0.409	0.466	0.495	0.520	0.565	0.582
	23		0.428	0.484	0.514	0.538	0.583	0.599
	24		0.446	0.502	0.532	0.556	0.601	0.617
	25		0.464	0.520	0.550	0.574	0.618	0.634

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.179	0.221	0.244	0.265	0.304	0.319
	12		0.194	0.237	0.261	0.282	0.322	0.337
	13		0.209	0.253	0.278	0.299	0.340	0.355
	14		0.225	0.270	0.285	0.316	0.357	0.373
	15		0.240	0.286	0.311	0.333	0.374	0.390
	16		0.255	0.302	0.328	0.350	0.391	0.407
	17		0.270	0.318	0.344	0.366	0.408	0.424
	18		0.286	0.334	0.360	0.382	0.425	0.440
	19		0.301	0.350	0.377	0.399	0.441	0.457
	20		0.316	0.366	0.393	0.415	0.457	0.473
	21		0.332	0.382	0.409	0.431	0.473	0.489
	22		0.347	0.397	0.424	0.447	0.489	0.505
	23		0.362	0.413	0.440	0.463	0.505	0.521
	24		0.378	0.429	0.456	0.478	0.521	0.536
	25		0.393	0.444	0.472	0.494	0.536	0.552
70	0		0.010	0.023	0.032	0.042	0.064	0.073
	1		0.024	0.042	0.054	0.066	0.091	0.101
	2		0.038	0.060	0.074	0.087	0.115	0.126
	3		0.052	0.077	0.093	0.107	0.136	0.148
	4		0.066	0.094	0.111	0.126	0.157	0.169
	5		0.081	0.111	0.128	0.144	0.177	0.189
	6		0.095	0.127	0.146	0.162	0.196	0.209
	7		0.109	0.143	0.162	0.180	0.214	0.227
	8		0.123	0.159	0.179	0.197	0.232	0.246
	9		0.137	0.174	0.195	0.214	0.250	0.264
	10		0.152	0.190	0.212	0.230	0.267	0.281
	11		0.166	0.205	0.228	0.247	0.284	0.299
	12		0.180	0.221	0.243	0.263	0.301	0.315
	13		0.194	0.236	0.259	0.279	0.318	0.332
	14		0.209	0.251	0.275	0.295	0.334	0.349
	15		0.223	0.266	0.290	0.311	0.350	0.365
	16		0.237	0.281	0.306	0.326	0.366	0.381
	17		0.251	0.296	0.321	0.342	0.382	0.397
	18		0.265	0.311	0.336	0.357	0.397	0.412
	19		0.280	0.326	0.351	0.372	0.413	0.428
	20		0.294	0.341	0.366	0.388	0.428	0.443
	21		0.308	0.356	0.381	0.403	0.443	0.458
	22		0.322	0.370	0.396	0.418	0.458	0.473
	23		0.337	0.385	0.411	0.432	0.473	0.488
	24		0.351	0.399	0.425	0.447	0.488	0.503
	25		0.365	0.414	0.440	0.462	0.502	0.518

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
75	0		0.009	0.021	0.030	0.039	0.060	0.068
	1		0.022	0.039	0.051	0.062	0.085	0.095
	2		0.035	0.056	0.069	0.082	0.107	0.118
	3		0.049	0.072	0.087	0.100	0.128	0.139
	4		0.062	0.088	0.104	0.118	0.147	0.159
	5		0.075	0.103	0.120	0.135	0.166	0.177
	6		0.089	0.119	0.136	0.152	0.183	0.196
	7		0.102	0.133	0.152	0.168	0.201	0.213
	8		0.115	0.148	0.167	0.184	0.218	0.231
	9		0.128	0.163	0.183	0.200	0.234	0.247
	10		0.142	0.177	0.198	0.216	0.251	0.264
	11		0.155	0.192	0.213	0.231	0.267	0.280
	12		0.168	0.206	0.228	0.246	0.283	0.296
	13		0.181	0.221	0.243	0.261	0.298	0.312
	14		0.195	0.235	0.257	0.276	0.314	0.328
	15		0.208	0.249	0.272	0.291	0.329	0.343
	16		0.221	0.263	0.286	0.306	0.344	0.358
	17		0.235	0.277	0.300	0.320	0.359	0.373
	18		0.248	0.291	0.315	0.335	0.373	0.388
	19		0.261	0.305	0.329	0.349	0.388	0.402
	20		0.274	0.319	0.343	0.363	0.402	0.417
	21		0.288	0.332	0.357	0.378	0.417	0.431
	22		0.301	0.346	0.371	0.392	0.431	0.446
	23		0.314	0.360	0.385	0.406	0.445	0.460
	24		0.327	0.374	0.399	0.420	0.459	0.474
	25		0.341	0.388	0.412	0.433	0.473	0.487
80	0		0.009	0.020	0.028	0.037	0.056	0.064
	1		0.021	0.037	0.048	0.058	0.080	0.089
	2		0.033	0.053	0.065	0.077	0.101	0.111
	3		0.046	0.068	0.082	0.094	0.120	0.131
	4		0.058	0.083	0.097	0.111	0.138	0.149
	5		0.071	0.097	0.113	0.127	0.156	0.167
	6		0.083	0.111	0.128	0.143	0.173	0.184
	7		0.095	0.125	0.143	0.158	0.189	0.201
	8		0.108	0.139	0.157	0.173	0.205	0.217
	9		0.120	0.153	0.172	0.188	0.221	0.233
	10		0.133	0.167	0.186	0.203	0.236	0.249
	11		0.145	0.180	0.200	0.217	0.251	0.264
	12		0.158	0.194	0.214	0.232	0.266	0.279
	13		0.170	0.207	0.228	0.246	0.281	0.294
	14		0.183	0.221	0.242	0.260	0.295	0.309
	15		0.195	0.233	0.256	0.274	0.310	0.323

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	16		0.207	0.247	0.269	0.288	0.324	0.338
	17		0.220	0.260	0.283	0.302	0.338	0.352
	18		0.232	0.273	0.296	0.315	0.352	0.366
	19		0.245	0.287	0.309	0.329	0.366	0.380
	20		0.257	0.300	0.323	0.342	0.380	0.394
	21		0.270	0.313	0.336	0.356	0.393	0.407
	22		0.282	0.326	0.349	0.369	0.407	0.421
	23		0.295	0.338	0.362	0.382	0.420	0.434
	24		0.307	0.351	0.375	0.395	0.433	0.447
	25		0.320	0.364	0.388	0.408	0.446	0.461
85	0		0.008	0.019	0.027	0.035	0.053	0.060
	1		0.020	0.035	0.045	0.055	0.076	0.084
	2		0.031	0.050	0.061	0.072	0.095	0.105
	3		0.043	0.064	0.077	0.089	0.113	0.123
	4		0.055	0.079	0.092	0.104	0.131	0.141
	5		0.066	0.091	0.106	0.120	0.147	0.158
	6		0.078	0.105	0.121	0.135	0.163	0.174
	7		0.090	0.118	0.135	0.149	0.178	0.190
	8		0.102	0.131	0.148	0.163	0.194	0.205
	9		0.113	0.144	0.162	0.177	0.208	0.220
	10		0.125	0.157	0.175	0.191	0.223	0.235
	11		0.137	0.170	0.189	0.205	0.237	0.250
	12		0.148	0.183	0.202	0.219	0.252	0.264
	13		0.160	0.195	0.215	0.232	0.266	0.278
	14		0.172	0.208	0.228	0.245	0.279	0.292
	15		0.184	0.221	0.241	0.259	0.293	0.306
	16		0.195	0.233	0.254	0.272	0.306	0.320
	17		0.207	0.245	0.267	0.285	0.320	0.333
	18		0.219	0.258	0.279	0.298	0.333	0.346
	19		0.230	0.270	0.292	0.311	0.346	0.360
	20		0.242	0.283	0.305	0.323	0.359	0.373
	21		0.254	0.295	0.317	0.336	0.372	0.386
	22		0.266	0.307	0.330	0.349	0.385	0.398
	23		0.277	0.319	0.342	0.361	0.398	0.411
	24		0.289	0.331	0.354	0.374	0.410	0.424
	25		0.301	0.344	0.367	0.386	0.423	0.436

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
90	0		0.008	0.018	0.025	0.033	0.050	0.057
	1		0.019	0.033	0.043	0.052	0.071	0.080
	2		0.030	0.047	0.058	0.068	0.090	0.099
	3		0.041	0.060	0.073	0.084	0.107	0.117
	4		0.052	0.074	0.087	0.099	0.124	0.133
	5		0.063	0.086	0.101	0.113	0.139	0.149
	6		0.074	0.099	0.114	0.127	0.154	0.165
	7		0.085	0.112	0.127	0.141	0.169	0.180
	8		0.096	0.124	0.140	0.155	0.183	0.195
	9		0.107	0.136	0.153	0.168	0.198	0.209
	10		0.118	0.149	0.166	0.181	0.211	0.223
	11		0.129	0.161	0.179	0.194	0.225	0.237
	12		0.140	0.173	0.191	0.207	0.238	0.250
	13		0.151	0.185	0.204	0.220	0.252	0.264
	14		0.162	0.197	0.216	0.232	0.265	0.277
	15		0.173	0.209	0.228	0.245	0.278	0.290
	16		0.185	0.220	0.240	0.257	0.291	0.303
	17		0.196	0.232	0.252	0.270	0.303	0.316
	18		0.207	0.244	0.265	0.282	0.316	0.329
	19		0.218	0.256	0.276	0.294	0.328	0.341
	20		0.229	0.267	0.288	0.306	0.341	0.354
	21		0.240	0.279	0.300	0.318	0.353	0.366
	22		0.251	0.290	0.312	0.330	0.365	0.378
	23		0.262	0.302	0.324	0.342	0.377	0.391
	24		0.273	0.314	0.336	0.354	0.389	0.403
	25		0.284	0.325	0.347	0.366	0.401	0.415
95	0		0.007	0.017	0.024	0.031	0.047	0.054
	1		0.018	0.031	0.040	0.049	0.068	0.076
	2		0.028	0.045	0.055	0.065	0.086	0.094
	3		0.039	0.057	0.069	0.080	0.102	0.111
	4		0.049	0.070	0.082	0.094	0.117	0.127
	5		0.059	0.082	0.095	0.107	0.132	0.142
	6		0.070	0.094	0.108	0.121	0.147	0.157
	7		0.080	0.106	0.121	0.134	0.161	0.171
	8		0.091	0.118	0.133	0.147	0.174	0.185
	9		0.101	0.129	0.145	0.159	0.188	0.199
	10		0.112	0.141	0.158	0.172	0.201	0.212

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.122	0.152	0.170	0.184	0.214	0.225
	12		0.133	0.164	0.181	0.197	0.227	0.238
	13		0.143	0.175	0.193	0.209	0.239	0.251
	14		0.154	0.187	0.205	0.221	0.252	0.264
	15		0.164	0.198	0.217	0.233	0.264	0.276
	16		0.175	0.209	0.228	0.245	0.276	0.288
	17		0.185	0.220	0.240	0.256	0.289	0.301
	18		0.196	0.231	0.251	0.268	0.301	0.313
	19		0.206	0.243	0.263	0.280	0.312	0.325
	20		0.217	0.254	0.274	0.391	0.324	0.337
	21		0.227	0.265	0.285	0.302	0.336	0.349
	22		0.238	0.275	0.296	0.314	0.348	0.360
	23		0.248	0.287	0.308	0.325	0.359	0.372
	24		0.259	0.298	0.319	0.337	0.371	0.383
	25		0.269	0.309	0.330	0.348	0.382	0.395
100	0		0.007	0.016	0.023	0.030	0.045	0.052
	1		0.017	0.030	0.038	0.047	0.065	0.072
	2		0.027	0.042	0.052	0.062	0.081	0.089
	3		0.037	0.054	0.066	0.076	0.097	0.105
	4		0.047	0.066	0.078	0.089	0.112	0.121
	5		0.057	0.078	0.091	0.102	0.126	0.135
	6		0.066	0.089	0.103	0.115	0.140	0.149
	7		0.076	0.101	0.115	0.127	0.153	0.163
	8		0.086	0.112	0.127	0.140	0.166	0.176
	9		0.096	0.123	0.138	0.152	0.179	0.189
	10		0.106	0.134	0.150	0.164	0.191	0.202
	11		0.116	0.145	0.161	0.176	0.204	0.215
	12		0.126	0.156	0.173	0.187	0.216	0.227
	13		0.136	0.167	0.184	0.199	0.228	0.239
	14		0.146	0.177	0.195	0.210	0.240	0.251
	15		0.156	0.188	0.206	0.222	0.252	0.263
	16		0.166	0.199	0.217	0.233	0.263	0.275
	17		0.176	0.210	0.228	0.244	0.275	0.287
	18		0.186	0.220	0.239	0.255	0.287	0.298
	19		0.196	0.231	0.250	0.266	0.298	0.310
	20		0.206	0.241	0.261	0.277	0.309	0.321
	21		0.216	0.252	0.271	0.288	0.320	0.333
	22		0.226	0.262	0.282	0.299	0.332	0.344
	23		0.236	0.273	0.293	0.310	0.343	0.355
	24		0.246	0.283	0.303	0.321	0.354	0.366
	25		0.256	0.294	0.314	0.331	0.365	0.377

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	26		0.266	0.304	0.325	0.342	0.375	0.388
	27		0.276	0.314	0.335	0.353	0.386	0.399
	28		0.286	0.325	0.346	0.363	0.397	0.409
	29		0.296	0.335	0.356	0.374	0.408	0.420
	30		0.306	0.345	0.366	0.384	0.418	0.431
	31		0.316	0.355	0.377	0.395	0.429	0.441
	32		0.326	0.366	0.387	0.405	0.439	0.452
	33		0.336	0.376	0.397	0.415	0.450	0.462
	34		0.346	0.386	0.408	0.426	0.460	0.472
	35		0.355	0.396	0.418	0.436	0.470	0.483
	36		0.365	0.406	0.428	0.446	0.481	0.493
	37		0.375	0.417	0.438	0.457	0.491	0.503
	38		0.385	0.427	0.449	0.467	0.501	0.513
	39		0.395	0.437	0.459	0.477	0.511	0.523
	40		0.405	0.447	0.469	0.487	0.521	0.534
	41		0.415	0.457	0.479	0.497	0.531	0.544
	42		0.425	0.467	0.489	0.507	0.541	0.553
	43		0.435	0.477	0.499	0.517	0.551	0.563
	44		0.445	0.487	0.509	0.527	0.561	0.573
	45		0.455	0.497	0.519	0.537	0.571	0.583
	46		0.465	0.507	0.529	0.547	0.581	0.593
	47		0.475	0.517	0.539	0.557	0.590	0.602
	48		0.485	0.527	0.549	0.567	0.600	0.612
	49		0.495	0.537	0.559	0.577	0.610	0.622
	50		0.505	0.547	0.569	0.586	0.619	0.631
110	0		0.006	0.015	0.021	0.027	0.041	0.047
	1		0.015	0.027	0.035	0.042	0.059	0.066
	2		0.024	0.038	0.048	0.056	0.074	0.082
	3		0.033	0.050	0.060	0.069	0.088	0.096
	4		0.042	0.060	0.071	0.081	0.102	0.110
	5		0.051	0.071	0.083	0.093	0.115	0.123
	6		0.060	0.081	0.094	0.105	0.127	0.136
	7		0.070	0.092	0.105	0.116	0.140	0.149
	8		0.079	0.102	0.115	0.127	0.152	0.161
	9		0.087	0.112	0.126	0.138	0.163	0.173
	10		0.097	0.122	0.137	0.149	0.175	0.185
	11		0.106	0.132	0.147	0.160	0.186	0.196
	12		0.115	0.142	0.157	0.171	0.197	0.208
	13		0.124	0.152	0.168	0.181	0.208	0.219
	14		0.133	0.162	0.178	0.192	0.219	0.230
	15		0.142	0.171	0.188	0.202	0.230	0.241

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	16		0.151	0.181	0.198	0.213	0.241	0.252
	17		0.160	0.191	0.208	0.223	0.252	0.262
	18		0.169	0.201	0.218	0.233	0.262	0.273
	19		0.178	0.210	0.228	0.243	0.273	0.284
	20		0.187	0.220	0.238	0.253	0.283	0.294
	21		0.196	0.229	0.248	0.263	0.293	0.305
	22		0.205	0.239	0.257	0.273	0.303	0.315
	23		0.215	0.249	0.267	0.283	0.314	0.325
	24		0.224	0.258	0.277	0.293	0.324	0.335
	25		0.233	0.268	0.287	0.303	0.334	0.345
	26		0.242	0.277	0.296	0.313	0.344	0.355
	27		0.251	0.286	0.306	0.322	0.354	0.365
	28		0.260	0.296	0.315	0.332	0.364	0.375
	29		0.269	0.305	0.325	0.342	0.373	0.385
	30		0.278	0.315	0.335	0.351	0.383	0.395
	31		0.287	0.324	0.344	0.361	0.393	0.405
	32		0.296	0.333	0.354	0.370	0.405	0.414
	33		0.305	0.343	0.363	0.380	0.412	0.424
	34		0.314	0.352	0.372	0.389	0.422	0.434
	35		0.323	0.361	0.382	0.399	0.431	0.443
	36		0.332	0.371	0.391	0.408	0.441	0.453
	37		0.341	0.380	0.401	0.418	0.450	0.462
	38		0.350	0.389	0.410	0.427	0.460	0.472
	39		0.360	0.398	0.419	0.436	0.469	0.481
	40		0.369	0.408	0.428	0.446	0.478	0.490
	41		0.378	0.417	9,438	0.455	0.488	0.500
	42		0.387	0.426	0.447	0.464	0.497	0.509
	43		0.396	0.435	0.456	0.474	0.506	0.518
	44		0.405	0.444	0.465	0.483	0.515	0.527
	45		0.414	0.454	0.475	0.492	0.524	0.536
	46		0.423	0.463	0.484	0.501	0.533	0.545
	47		0.432	0.472	0.493	0.510	0.543	0.554
	48		0.441	0.481	0.502	0.519	0.552	0.563
	49		0.450	0.490	0.511	0.528	0.561	0.572
	50		0.459	0.499	0.520	0.537	0.569	0.581
120	0		0.006	0.013	0.019	0.025	0.038	0.043
	1		0.014	0.025	0.032	0.039	0.054	0.060
	2		0.022	0.035	0.044	0.052	0.068	0.075
	3		0.031	0.045	0.055	0.063	0.081	0.088
	4		0.038	0.055	0.066	0.075	0.094	0.101
	5		0.047	0.065	0.076	0.086	0.106	0.113

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	6		0.055	0.075	0.086	0.096	0.117	0.125
	7		0.064	0.084	0.096	0.107	0.128	0.137
	8		0.072	0.093	0.107	0.117	0.139	0.148
	9		0.080	0.103	0.116	0.127	0.150	0.159
	10		0.089	0.112	0.125	0.137	0.161	0.170
	11		0.097	0.121	0.135	0.147	0.171	0.181
	12		0.105	0.130	0.145	0.157	0.182	0.191
	13		0.114	0.139	0.154	0.167	0.192	0.202
	14		0.122	0.148	0.163	0.176	0.202	0.212
	15		0.130	0.157	0.173	0.186	0.212	0.222
	16		0.139	0.166	0.182	0.195	0.222	0.232
	17		0.147	0.175	0.191	0.205	0.232	0.242
	18		0.155	0.184	0.200	0.214	0.241	0.252
	19		0.163	0.193	0.210	0.224	0.251	0.262
	20		0.172	0.202	0.219	0.233	0.261	0.271
	21		0.180	0.211	0.228	0.242	0.270	0.281
	22		0.186	0.220	0.237	0.251	0.280	0.290
	23		0.197	0.228	0.246	0.260	0.289	0.300
	24		0.205	0.237	0.255	0.270	0.298	0.309
	25		0.213	0.246	0.264	0.279	0.308	0.319
	26		0.222	0.254	0.272	0.288	0.317	0.328
	27		0.230	0.263	0.281	0.297	0.326	0.337
	28		0.238	0.272	0.290	0.306	0.335	0.346
	29		0.247	0.281	0.299	0.315	0.344	0.356
	30		0.255	0.289	0.308	0.323	0.354	0.365
	31		0.263	0.298	0.317	0.332	0.363	0.374
	32		0.271	0.306	0.325	0.341	0.372	0.383
	33		0.280	0.315	0.334	0.350	0.380	0.392
	34		0.288	0.324	0.343	0.359	0.389	0.401
	35		0.296	0.332	0.351	0.368	0.398	0.410
	36		0.305	0.341	0.360	0.376	0.407	0.418
	37		0.313	0.349	0.369	0.385	0.416	0.427
	38		0.321	0.358	0.377	0.394	0.425	0.436
	39		0.330	0.366	0.386	0.402	0.433	0.445
	40		0.338	0.375	0.394	0.411	0.442	0.453
	41		0.346	0.383	0.403	0.419	0.451	0.462
	42		0.355	0.392	0.412	0.428	0.459	0.471
	43		0.363	0.400	0.420	0.437	0.468	0.479
	44		0.371	0.409	0.429	0.445	0.476	0.488
	45		0.380	0.417	0.437	0.454	0.485	0.496
	46		0.388	0.426	0.446	0.462	0.493	0.505
	47		0.396	0.434	0.454	0.471	0.502	0.513
	48		0.404	0.442	0.462	0.479	0.510	0.522
	49		0.412	0.451	0.471	0.487	0.519	0.530
	50		0.421	0.459	0.479	0.496	0.527	0.538

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
130	0		0.005	0.012	0.018	0.023	0.035	0.040
	1		0.013	0.023	0.030	0.036	0.050	0.056
	2		0.021	0.033	0.040	0.048	0.063	0.069
	3		0.028	0.042	0.051	0.059	0.075	0.082
	4		0.036	0.051	0.061	0.069	0.087	0.094
	5		0.044	0.060	0.070	0.079	0.098	0.105
	6		0.051	0.069	0.080	0.089	0.108	0.116
	7		0.059	0.078	0.089	0.099	0.119	0.127
	8		0.067	0.086	0.098	0.108	0.129	0.137
	9		0.074	0.095	0.107	0.118	0.139	0.147
	10		0.082	0.104	0.116	0.127	0.149	0.157
	11		0.090	0.112	0.125	0.136	0.159	0.167
	12		0.097	0.120	0.134	0.145	0.168	0.177
	13		0.105	0.129	0.142	0.154	0.178	0.187
	14		0.113	0.137	0.151	0.163	0.187	0.196
	15		0.120	0.146	0.160	0.172	0.196	0.206
	16		0.128	0.154	0.168	0.181	0.206	0.215
	17		0.136	0.162	0.177	0.190	0.215	0.224
	18		0.143	0.170	0.185	0.198	0.224	0.234
	19		0.151	0.178	0.194	0.207	0.233	0.243
	20		0.159	0.187	0.202	0.216	0.242	0.252
	21		0.166	0.195	0.211	0.224	0.251	0.261
	22		0.174	0.203	0.219	0.233	0.259	0.269
	23		0.182	0.211	0.227	0.241	0.268	0.278
	24		0.189	0.219	0.236	0.250	0.277	0.287
	25		0.197	0.227	0.244	0.258	0.286	0.296
	26		0.205	0.235	0.252	0.267	0.294	0.305
	27		0.212	0.243	0.260	0.275	0.303	0.313
	28		0.220	0.251	0.269	0.283	0.311	0.322
	29		0.228	0.259	0.277	0.291	0.320	0.330
	30		0.235	0.267	0.285	0.300	0.328	0.338
	31		0.243	0.275	0.293	0.308	0.337	0.347
	32		0.251	0.283	0.301	0.316	0.345	0.355
	33		0.258	0.291	0.309	0.324	0.353	0.363
	34		0.266	0.299	0.317	0.333	0.362	0.372
	35		0.274	0.307	0.325	0.341	0.370	0.380
	36		0.281	0.315	0.333	0.349	0.378	0.388
	37		0.289	0.323	0.341	0.357	0.386	0.397
	38		0.297	0.331	0.349	0.365	0.394	0.405
	39		0.304	0.339	0.357	0.373	0.403	0.413
	40		0.312	0.347	0.365	0.381	0.411	0.421

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	41		0.320	0.355	0.373	0.389	0.419	0.429
	42		0.327	0.363	0.381	0.397	0.427	0.437
	43		0.335	0.370	0.389	0.405	0.435	0.445
	44		0.343	0.378	0.397	0.413	0.443	0.453
	45		0.350	0.386	0.405	0.421	0.451	0.461
	46		0.358	0.394	0.413	0.429	0.459	0.469
	47		0.366	0.402	0.421	0.437	0.467	0.477
	48		0.373	0.409	0.429	0.444	0.474	0.485
	49		0.381	0.417	0.436	0.452	0.482	0.493
	50		0.389	0.425	0.444	0.460	0.490	0.501
140	0		0.005	0.011	0.016	0.021	0.032	0.037
	1		0.012	0.021	0.027	0.033	0.046	0.052
	2		0.019	0.030	0.038	0.044	0.059	0.065
	3		0.026	0.039	0.047	0.054	0.070	0.076
	4		0.033	0.048	0.056	0.064	0.081	0.087
	5		0.040	0.056	0.065	0.074	0.091	0.098
	6		0.048	0.064	0.074	0.083	0.101	0.108
	7		0.055	0.072	0.083	0.092	0.111	0.118
	8		0.062	0.080	0.091	0.101	0.120	0.128
	9		0.069	0.088	0.100	0.110	0.130	0.137
	10		0.076	0.096	0.108	0.118	0.139	0.147
	11		0.083	0.104	0.116	0.127	0.148	0.156
	12		0.090	0.112	0.124	0.135	0.157	0.165
	13		0.097	0.120	0.133	0.144	0.166	0.174
	14		0.105	0.128	0.141	0.152	0.174	0.183
	15		0.112	0.135	0.149	0.160	0.183	0.192
	16		0.119	0.143	0.157	0.168	0.192	0.200
	17		0.126	0.151	0.165	0.177	0.200	0.209
	18		0.133	0.158	0.173	0.185	0.209	0.218
	19		0.140	0.166	0.180	0.192	0.217	0.226
	20		0.147	0.174	0.188	0.201	0.225	0.235
	21		0.154	0.181	0.196	0.209	0.234	0.243
	22		0.162	0.189	0.204	0.217	0.242	0.251
	23		0.169	0.196	0.212	0.225	0.250	0.260
	24		0.176	0.204	0.219	0.233	0.258	0.268
	25		0.183	0.211	0.227	0.240	0.266	0.276
	26		0.190	0.219	0.235	0.248	0.274	0.284
	27		0.197	0.226	0.242	0.256	0.282	0.292
	28		0.204	0.234	0.250	0.264	0.290	0.300
	29		0.211	0.241	0.258	0.272	0.298	0.308
	30		0.219	0.249	0.265	0.279	0.306	0.316

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	31		0.226	0.256	0.273	0.287	0.314	0.324
	32		0.233	0.264	0.280	0.295	0.322	0.332
	33		0.240	0.271	0.288	0.302	0.330	0.340
	34		0.247	0.278	0.296	0.310	0.337	0.348
	35		0.254	0.286	0.303	0.317	0.345	0.355
	36		0.261	0.293	0.311	0.325	0.353	0.363
	37		0.268	0.301	0.318	0.333	0.361	0.371
	38		0.276	0.308	0.325	0.340	0.368	0.379
	39		0.283	0.315	0.333	0.348	0.376	0.386
	40		0.290	0.323	0.340	0.355	0.383	0.394
	41		0.297	0.330	0.348	0.363	0.391	0.401
	42		0.304	0.337	0.355	0.370	0.398	0.409
	43		0.311	0.345	0.363	0.378	0.406	0.417
	44		0.318	0.352	0.370	0.385	0.414	0.424
	45		0.325	0.359	0.377	0.392	0.421	0.432
	46		0.333	0.367	0.385	0.400	0.428	0.439
	47		0.340	0.374	0.392	0.407	0.436	0.446
	48		0.347	0.381	0.399	0.414	0.443	0.454
	49		0.354	0.388	0.407	0.422	0.451	0.461
	50		0.361	0.396	0.414	0.429	0.458	0.469
150	0		0.005	0.011	0.015	0.020	0.030	0.035
	1		0.011	0.020	0.026	0.031	0.043	0.048
	2		0.018	0.028	0.035	0.041	0.055	0.060
	3		0.024	0.036	0.044	0.051	0.065	0.071
	4		0.031	0.044	0.053	0.060	0.075	0.082
	5		0.038	0.052	0.061	0.069	0.085	0.091
	6		0.044	0.060	0.069	0.077	0.094	0.101
	7		0.051	0.067	0.077	0.086	0.104	0.111
	8		0.058	0.075	0.085	0.094	0.112	0.120
	9		0.064	0.082	0.093	0.102	0.121	0.128
	10		0.071	0.090	0.101	0.110	0.130	0.137
	11		0.078	0.097	0.109	0.118	0.138	0.146
	12		0.084	0.105	0.116	0.126	0.147	0.155
	13		0.091	0.112	0.124	0.134	0.155	0.163
	14		0.098	0.119	0.131	0.142	0.163	0.171
	15		0.104	0.126	0.139	0.150	0.171	0.180
	16		0.111	0.134	0.146	0.157	0.179	0.188
	17		0.118	0.141	0.154	0.165	0.187	0.196
	18		0.124	0.148	0.161	0.173	0.195	0.204
	19		0.131	0.155	0.169	0.180	0.203	0.212
	20		0.137	0.162	0.176	0.188	0.211	0.220

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	21		0.144	0.169	0.183	0.195	0.219	0.228
	22		0.151	0.176	0.191	0.203	0.227	0.236
	23		0.157	0.183	0.198	0.210	0.234	0.243
	24		0.164	0.191	0.205	0.218	0.242	0.251
	25		0.171	0.198	0.212	0.225	0.249	0.259
	26		0.177	0.205	0.220	0.232	0.257	0.266
	27		0.184	0.212	0.227	0.240	0.265	0.274
	28		0.191	0.219	0.234	0.247	0.272	0.281
	29		0.197	0.226	0.241	0.254	0.279	0.289
	30		0.204	0.233	0.248	0.261	0.287	0.296
	31		0.211	0.239	0.255	0.269	0.294	0.304
	32		0.217	0.246	0.262	0.276	0.302	0.311
	33		0.224	0.253	0.269	0.283	0.309	0.319
	34		0.231	0.260	0.276	0.290	0.316	0.326
	35		0.237	0.267	0.283	0.297	0.324	0.333
	36		0.244	0.274	0.291	0.304	0.331	0.341
	37		0.251	0.281	0.298	0.311	0.338	0.348
	38		0.257	0.288	0.305	0.318	0.345	0.355
	39		0.264	0.295	0.311	0.326	I,352	0.362
	40		0.271	0.302	0.318	0.333	0.360	0.370
	41		0.277	0.309	0.325	0.340	0.367	0.377
	42		0.284	0.315	0.332	0.347	0.374	0.384
	43		0.290	0.322	0.339	0.354	0.381	0.391
	44		0.297	0.329	0.346	0.361	0.388	0.398
	45		0.304	0.336	0.353	0.368	0.395	0.405
	46		0.310	0.343	0.360	0.374	0.402	0.412
	47		0.317	0.350	0.367	0.381	0.409	0.419
	48		0.324	0.356	0.373	0.388	0.416	0.426
	49		0.330	0.363	0.381	0.395	0.423	0.433
	50		0.337	0.370	0.387	0.402	0.430	0.440
160	0		0.004	0.010	0.014	0.019	0.028	0.033
	1		0.010	0.019	0.024	0.029	0.041	0.046
	2		0.017	0.027	0.033	0.039	0.051	0.057
	3		0.023	0.034	0.041	0.048	0.061	0.067
	4		0.029	0.042	0.049	0.056	0.071	0.077
	5		0.035	0.049	0.057	0.065	0.080	0.086

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	6		0.042	0.056	0.065	0.073	0.089	0.095
	7		0.048	0.063	0.072	0.081	0.097	0.104
	8		0.054	0.070	0.080	0.088	0.106	0.112
	9		0.060	0.077	0.087	0.096	0.114	0.121
	10		0.067	0.084	0.095	0.104	0.122	0.129
	11		0.073	0.091	0.102	0.111	0.130	0.137
	12		0.079	0.098	0.109	0.119	0.138	0.145
	13		0.085	0.105	0.116	0.126	0.146	0.153
	14		0.091	0.112	0.123	0.133	0.153	0.161
	15		0.098	0.119	0.130	0.141	0.161	0.169
	16		0.104	0.125	0.137	0.148	0.169	0.177
	17		0.110	0.132	0.144	0.155	0.176	0.184
	18		0.166	0.139	0.151	0.162	0.184	0.192
	19		0.123	0.146	0.158	0.169	0.191	0.199
	20		0.129	0.152	0.165	0.176	0.198	0.207
	21		0.135	0.159	0.172	0.183	0.206	0.214
	22		0.141	0.166	0.179	0.190	0.213	0.222
	23		0.148	0.172	0.186	0.197	0.220	0.229
	24		0.154	0.179	0.193	0.204	0.227	0.236
	25		0.160	0.185	0.199	0.211	0.235	0.243
	26		0.16?	0.192	0.206	0.218	0.242	0.251
	27		0.173	0.199	0.213	0.225	0.249	0.258
	28		0.179	0.205	0.220	0.232	0.256	0.265
	29		0.185	0.212	0.226	0.239	0.263	0.272
	30		0.191	0.218	0.233	0.246	0.270	0.279
	31		0.198	0.225	0.240	0.252	0.277	0.286
	32		0.204	0.231	0.246	0.259	0.284	0.293
	33		0.210	0.238	0.253	0.266	0.291	0.300
	34		0.216	0.244	0.260	0.273	0.298	0.307
	35		0.222	0.251	0.266	0.279	0.304	0.314
	36		0.229	0.257	0.273	0.286	0.311	0.321
	37		0.235	0.264	0.280	0.293	0.318	0.328
	38		0.241	0.270	0.286	0.299	0.325	0.334
	39		0.247	0.277	0.293	0.306	0.332	0.341
	40		0.254	0.283	0.299	0.313	0.338	0.348
	41		0.260	0.290	0.306	0.319	0.345	0.355
	42		0.266	0.296	0.312	0.326	0.352	0.362
	43		0.272	0.303	0.319	0.332	0.359	0.368
	44		0.279	0.309	0.325	0.339	0.365	0.375
	45		0.285	0.315	0.332	0.346	0.372	0.382
	46		0.291	0.322	0.338	0.352	0.378	0.388
	47		0.297	0.328	0.345	0.359	0.385	0.395
	48		0.304	0.335	0.351	0.365	0.392	0.401
	49		0.310	0.341	0.358	0.372	0.398	0.408
	50		0.316	0.347	0.364	0.378	0.405	0.415

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
170	0		0.004	0.009	0.013	0.017	0.027	0.031
	1		0.010	0.018	0.023	0.028	0.038	0.043
	2		0.016	0.025	0.031	0.037	0.049	0.053
	3		0.022	0.032	0.039	0.045	0.058	0.063
	4		0.027	0.039	0.046	0.053	0.069	0.072
	5		0.033	0.046	0.054	0.061	0.075	0.081
	6		0.039	0.053	0.061	0.068	0.084	0.090
	7		0.045	0.060	0.068	0.076	0.092	0.098
	8		0.051	0.066	0.075	0.083	0.100	0.106
	9		0.057	0.073	0.082	0.091	0.107	0.114
	10		0.063	0.079	0.089	0.098	0.115	0.122
	11		0.069	0.086	0.096	0.105	0.123	0.129
	12		0.074	0.092	0.103	0.112	0.130	0.137
	13		0.080	0.099	0.110	0.119	0.137	0.145
	14		0.086	0.105	0.116	0.126	0.145	0.152
	15		0.092	0.112	0.123	0.133	0.152	0.159
	16		0.098	0.118	0.130	0.139	0.159	0.167
	17		0.104	0.124	0.136	0.146	0.166	0.174
	18		0.110	0.131	0.143	0.153	0.173	0.181
	19		0.115	0.137	0.149	0.160	0.180	0.188
	20		0.121	0.143	0.156	0.166	0.187	0.195
	21		0.127	0.150	0.162	0.173	0.194	0.202
	22		0.133	0.156	0.169	0.180	0.201	0.209
	23		0.139	0.162	0.175	0.186	0.208	0.216
	24		0.145	0.168	0.182	0.193	0.215	0.223
	25		0.151	0.175	0.188	0.199	0.221	0.230
	26		0.157	0.181	0.194	0.206	0.228	0.237
	27		0.162	0.187	0.201	0.212	0.235	0.243
	28		0.168	0.193	0.207	0.219	0.242	0.250
	29		0.174	0.199	0.213	0.225	0.248	0.257
	30		0.180	0.206	0.220	0.232	0.255	0.264
	31		0.186	0.212	0.226	0.238	0.261	0.270
	32		0.192	0.218	0.232	0.244	0.268	0.277
	33		0.198	0.224	0.239	0.251	0.275	0.283
	34		0.204	0.230	0.245	0.257	0.281	0.290
	35		0.209	0.236	0.251	0.264	0.288	0.296
	36		0.215	0.243	0.257	0.270	0.294	0.303
	37		0.221	0.249	0.264	0.276	0.300	0.310
	38		0.227	0.255	0.270	0.282	0.307	0.316
	39		0.233	0.261	0.276	0.289	0.313	0.322
	40		0.239	0.267	0.282	0.295	0.320	0.329

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	41		0.245	0.273	0.288	0.301	0.326	0.335
	42		0.250	0.279	0.295	0.308	0.332	0.342
	43		0.256	0.285	0.301	0.314	0.339	0.348
	44		0.262	0.291	0.307	0.320	0.345	0.354
	45		0.268	0.297	0.313	0.326	0.351	0.361
	46		0.274	0.303	0.319	0.332	0.358	0.367
	47		0.280	0.309	0.325	0.339	0.364	0.373
	48		0.286	0.315	0.331	0.345	0.370	0.380
	49		0.292	0.321	0.337	0.351	0.376	0.386
	50		0.297	0.327	0.344	0.357	0.383	0.392
180	0		0.004	0.009	0.013	0.017	0.025	0.029
	1		0.009	0.017	0.021	0.026	0.036	0.041
	2		0.015	0.024	0.029	0.035	0.046	0.050
	3		0.020	0.030	0.037	0.043	0.054	0.060
	4		0.026	0.037	0.044	0.050	0.063	0.068
	5		0.031	0.044	0.051	0.058	0.071	0.077
	6		0.037	0.050	0.058	0.065	0.079	0.085
	7		0.043	0.056	0.065	0.072	0.087	0.093
	8		0.048	0.063	0.071	0.079	0.094	0.100
	9		0.054	0.069	0.078	0.086	0.102	0.108
	10		0.059	0.075	0.084	0.092	0.109	0.115
	11		0.065	0.081	0.091	0.099	0.116	0.122
	12		0.070	0.087	0.097	0.106	0.123	0.130
	13		0.076	0.093	0.104	0.112	0.130	0.137
	14		0.081	0.100	0.110	0.119	0.137	0.144
	15		0.087	0.106	0.116	0.125	0.144	0.151
	16		0.092	0.112	0.122	0.132	0.151	0.158
	17		0.098	0.118	0.129	0.138	0.157	0.165
	18		0.104	0.124	0.135	0.145	0.164	0.171
	19		0.109	0.130	0.141	0.151	0.171	0.178
	20		0.115	0.136	0.147	0.157	0.177	0.185
	21		0.120	0.141	0.153	0.164	0.184	0.191
	22		0.126	0.147	0.160	0.170	0.190	0.198
	23		0.131	0.153	0.166	0.176	0.197	0.205
	24		0.137	0.159	0.172	0.182	0.203	0.211
	25		0.142	0.165	0.178	0.189	0.210	0.218
	26		0.148	0.171	0.184	0.195	0.216	0.224
	27		0.153	0.177	0.190	0.201	0.222	0.231

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	28		0.159	0.183	0.196	0.207	0.229	0.237
	29		0.165	0.189	0.202	0.213	0.235	0.243
	30		0.170	0.194	0.208	0.219	0.241	0.250
	31		0.176	0.200	0.214	0.225	0.248	0.256
	32		0.181	0.206	0.220	0.231	0.254	0.262
	33		0.187	0.212	0.226	0.237	0.260	0.268
	34		0.192	0.218	0.232	0.243	0.266	0.275
	35		0.198	0.224	0.238	0.249	0.272	0.281
	36		0.203	0.229	0.243	0.255	0.278	0.287
	37		0.209	0.235	0.249	0.261	0.285	0.293
	38		0.214	0.241	0.255	0.267	0.291	0.299
	39		0.220	0.247	0.261	0.273	0.297	0.306
	40		0.226	0.252	0.267	0.279	0.303	0.312
	41		0.231	0.258	0.273	0.285	0.309	0.318
	42		0.237	0.264	0.279	0.291	0.315	0.324
	43		0.242	0.270	0.284	0.297	0.321	0.330
	44		0.248	0.275	0.290	0.303	0.327	0.336
	45		0.253	0.281	0.296	0.309	0.333	0.342
	46		0.259	0.287	0.302	0.315	0.339	0.348
	47		0.264	0.293	0.308	0.320	0.345	0.354
	48		0.270	0.298	0.314	0.326	0.351	0.360
	49		0.275	0.304	0.319	0.332	0.357	0.366
	50		0.281	0.310	0.325	0.338	0.363	0.372
190	0		0.004	0.008	0.012	0.016	0.024	0.028
	1		0.009	0.016	0.020	0.025	0.034	0.038
	2		0.014	0.022	0.028	0.033	0.044	0.048
	3		0.019	0.029	0.035	0.040	0.052	0.057
	4		0.025	0.035	0.042	0.048	0.060	0.065
	5		0.030	0.041	0.048	0.055	0.068	0.073
	6		0.035	0.047	0.055	0.061	0.075	0.080
	7		0.040	0.053	0.061	0.068	0.082	0.088
	8		0.046	0.059	0.067	0.075	0.089	0.095
	9		0.051	0.065	0.074	0.081	0.096	0.109
	10		0.056	0.071	0.080	0.088	0.103	0.109
	11		0.061	0.077	0.086	0.094	0.110	0.116
	12		0.067	0.083	0.092	0.100	0.117	0.123
	13		0.072	0.089	0.098	0.107	0.123	0.130
	14		0.077	0.094	0.104	0.113	0.130	0.136
	15		0.082	0.100	0.110	0.119	0.136	0.143
	16		0.088	0.106	0.116	0.125	0.143	0.150
	17		0.093	0.112	0.122	0.131	0.149	0.156
	18		0.098	0.117	0.128	0.137	0.156	0.163
	19		0.103	0.123	0.134	0.143	0.162	0.169
	20		0.109	0.129	0.140	0.149	0.168	0.175

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	21		0.114	0.134	0.146	0.155	0.174	0.182
	22		0.119	0.140	9.151	0.161	0.181	0.188
	23		0.124	0.145	0.157	0.167	0.187	0.194
	24		0.130	0.151	0.163	0.173	0.193	0.200
	25		0.135	0.157	0.169	0.179	0.199	0.207
	26		0.140	0.162	0.174	0.185	0.205	0.213
	27		0.145	0.167	0.180	0.191	0.211	0.219
	28		0.151	0.173	0.186	0.196	0.217	0.225
	29		0.156	0.179	0.191	0.202	0.223	0.231
	30		0.161	0.184	0.197	0.208	0.229	0.237
	31		0.166	0.190	0.203	0.214	0.235	0.243
	32		0.172	0.195	0.208	0.220	2.41	0.249
	33		0.177	0.201	0.214	0.225	0.247	0.255
	34		0.182	0.206	0.220	0.231	0.253	0.261
	35		0.187	0.212	0.225	0.237	0.259	0.267
	36		0.193	0.217	0.231	0.242	0.265	0.273
	37		0.198	0.223	0.237	0.248	0.270	0.279
	38		0.203	0.228	0.242	0.254	0.276	0.285
	39		0.208	0.234	0.248	0.259	0.282	0.290
	40		0.214	0.239	0.253	0.265	0.288	0.296
	41		0.219	0.245	0.259	0.271	0.293	0.302
	42		0.224	0.250	0.264	0.276	0.299	0.308
	43		0.229	0.256	0.270	0.282	0.305	0.314
	44		0.235	0.261	0.275	0.288	0.311	0.319
	45		0.240	0.267	0.281	0.293	0.316	0.325
	46		0.245	0.272	0.287	0.299	0.322	0.331
	47		0.250	0.277	0.292	0.304	0.328	0.336
	48		0.256	0.283	0.298	0.310	0.333	0.342
	49		0.261	0.288	0.303	0.315	0.339	0.348
	50		0.266	0.294	0.309	0.321	0.345	0.353
200	0		0.003	0.008	0.011	0.015	0.023	0.026
	1		0.008	0.015	0.019	0.023	0.033	0.037
	2		0.013	0.021	0.026	0.031	0.041	0.046
	3		0.018	0.027	0.033	0.038	0.049	0.054
	4		0.023	0.033	0.040	0.045	0.057	0.062
	5		0.028	0.039	0.046	0.052	0.064	0.069
	6		0.033	0.045	0.052	0.058	0.071	0.075
	7		0.038	0.051	0.058	0.065	0.078	0.084
	8		0.043	0.056	0.064	0.071	0.085	0.090
	9		0.048	0.062	0.070	0.077	0.092	0.097
	10		0.053	0.068	0.076	0.083	0.098	0.104

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

$\underline{N}$	$\underline{F}$	$\underline{C}$	.500	.800	.900	.950	.990	.995
	11		0.058	0.073	0.082	0.089	0.105	0.111
	12		0.063	0.079	0.088	0.095	0.111	0.117
	13		0.068	0.084	0.093	0.101	0.117	0.124
	14		0.073	0.090	0.099	0.107	0.124	0.130
	15		0.078	0.095	0.105	0.113	0.130	0.136
	16		0.083	0.101	0.110	0.119	0.136	0.142
	17		0.088	0.106	0.116	0.125	0.142	0.149
	18		0.093	0.111	0.122	0.131	0.148	0.155
	19		0.098	0.117	0.127	0.136	0.154	0.161
	20		0.103	0.122	0.133	0.142	0.160	0.167
	21		0.108	0.128	0.138	0.148	0.166	0.173
	22		0.113	0.133	0.144	0.153	0.172	0.179
	23		0.118	0.138	0.149	0.159	0.178	0.185
	24		0.123	0.144	0.155	0.164	0.184	0.191
	25		0.128	0.149	0.160	0.170	0.189	0.197
	26		0.133	0.154	0.166	0.176	0.195	0.203
	27		0.138	0.159	0.171	0.181	0.201	0.208
	28		0.143	0.165	0.177	0.187	0.207	0.214
	29		0.148	0.170	0.182	0.192	0.212	0.220
	30		0.153	0.175	0.188	0.198	0.218	0.226
	31		0.158	0.181	0.193	0.203	0.224	0.232
	32		0.163	0.186	0.198	0.209	0.229	0.237
	33		0.168	0.191	0.204	0.214	0.235	0.243
	34		0.173	0.196	0.209	0.220	0.241	0.249
	35		0.178	0.202	0.214	0.225	0.246	0.254
	36		0.183	0.207	0.220	0.231	0.252	0.260
	37		0.188	0.212	0.225	0.236	0.257	0.265
	38		0.193	0.217	0.230	0.241	0.263	0.271
	39		0.198	0.222	0.236	0.247	0.269	0.277
	40		0.203	0.228	0.240	0.252	0.274	0.282
	41		0.208	0.233	0.246	0.258	0.280	0.288
	42		0.213	0.238	0.252	0.263	0.285	0.293
	43		0.218	0.243	0.257	0.268	0.290	0.299
	44		0.223	0.248	0.262	0.274	0.296	0.304
	45		0.228	0.254	0.267	0.279	0.301	0.310
	46		0.233	0.259	0.273	0.284	0.307	0.315
	47		0.238	0.264	0.278	0.290	0.312	0.321
	48		0.243	0.269	0.283	0.295	0.318	0.326
	49		0.248	0.274	0.288	0.300	0.323	0.331
	50		0.253	0.279	0.294	0.305	0.328	0.337

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
210	0		0.003	0.008	0.011	0.014	0.022	0.025
	1		0.008	0.014	0.018	0.022	0.031	0.035
	2		0.013	0.020	0.025	0.030	0.039	0.043
	3		0.017	0.026	0.032	0.037	0.047	0.051
	4		0.022	0.032	0.038	0.043	0.054	0.059
	5		0.027	0.037	0.044	0.049	0.061	0.066
	6		0.032	0.043	0.050	0.056	0.068	0.073
	7		0.036	0.048	0.055	0.062	0.075	0.080
	8		0.041	0.054	0.061	0.068	0.081	0.086
	9		0.046	0.059	0.067	0.074	0.087	0.091
	10		0.051	0.064	0.072	0.079	0.094	0.099
	11		0.055	0.070	0.078	0.085	0.100	0.105
	12		0.060	0.075	0.084	0.091	0.106	0.112
	13		0.065	0.080	0.089	0.097	0.112	0.118
	14		0.070	0.085	0.094	0.102	0.118	0.124
	15		0.074	0.091	0.100	0.108	0.124	0.130
	16		0.079	0.096	0.105	0.113	0.130	0.136
	17		0.084	0.101	0.111	0.119	0.135	0.142
	18		0.089	0.106	0.116	0.124	0.141	0.148
	19		0.094	0.111	0.121	0.130	0.147	0.154
	20		0.098	0.116	0.127	0.135	0.153	0.159
	21		0.103	0.122	0.132	0.141	0.158	0.165
	22		0.108	0.127	0.137	0.146	0.164	0.171
	23		0.113	0.132	0.142	0.152	0.170	0.176
	24		0.117	0.137	0.147	0.157	0.175	0.182
	25		0.122	0.142	0.153	0.162	0.181	0.188
	26		0.127	0.147	0.158	0.168	0.186	0.193
	27		0.132	0.152	0.163	0.173	0.192	0.199
	28		0.136	0.157	0.168	0.178	0.197	0.204
	29		0.141	0.162	0.174	0.183	0.203	0.210
	30		0.146	0.167	0.179	0.189	0.208	0.216
	31		0.151	0.172	0.184	0.194	0.214	0.221
	32		0.155	0.177	0.189	0.199	0.219	0.226
	33		0.160	0.182	0.194	0.204	0.224	0.232
	34		0.165	0.187	0.199	0.210	0.230	0.237
	35		0.170	0.192	0.204	0.215	0.235	0.243
	36		0.174	0.197	0.209	0.220	0.240	0.248
	37		0.179	0.202	0.215	0.225	0.246	0.253
	38		0.184	0.207	0.220	0.230	0.251	0.259
	39		0.189	0.212	0.225	0.235	0.256	0.264
	40		0.193	0.217	0.230	0.241	0.262	0.269

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	41		0.198	0.222	0.235	0.246	0.267	0.275
	42		0.203	0.227	0.240	0.251	0.272	0.280
	43		0.208	0.232	0.245	0.256	0.277	0.285
	44		0.212	0.237	0.250	0.261	0.282	0.291
	45		0.217	0.242	0.255	0.266	0.288	0.296
	46		0.222	0.247	0.260	0.271	0.293	0.301
	47		0.227	0.252	0.265	0.276	0.298	0.306
	48		0.231	0.256	0.270	0.281	0.303	0.311
	49		0.236	0.261	0.275	0.286	0.308	0.317
	50		0.241	0.266	0.280	0.291	0.313	0.322
220	0		0.003	0.007	0.010	0.014	0.021	0.024
	1		0.008	0.013	0.018	0.021	0.030	0.033
	2		0.012	0.019	0.024	0.028	0.038	0.041
	3		0.017	0.025	0.030	0.035	0.045	0.049
	4		0.021	0.030	0.036	0.041	0.052	0.056
	5		0.026	0.036	0.042	0.047	0.058	0.063
	6		0.030	0.041	0.047	0.053	0.065	0.070
	7		0.035	0.046	0.053	0.059	0.071	0.076
	8		0.039	0.051	0.058	0.065	0.077	0.082
	9		0.044	0.056	0.064	0.070	0.083	0.089
	10		0.048	0.062	0.069	0.076	0.089	0.095
	11		0.053	0.067	0.074	0.081	0.095	0.101
	12		0.057	0.072	0.080	0.087	0.101	0.107
	13		0.062	0.077	0.085	0.092	0.107	0.113
	14		0.067	0.082	0.090	0.098	0.113	0.118
	15		0.071	0.087	0.095	0.103	0.118	0.124
	16		0.076	0.092	0.101	0.108	0.124	0.130
	17		0.080	0.096	0.106	0.114	0.130	0.136
	18		0.085	0.101	0.111	0.119	0.135	0.141
	19		0.089	0.106	0.116	0.124	0.141	0.147
	20		0.094	0.111	0.121	0.129	0.146	0.152
	21		0.098	0.116	0.126	0.135	0.151	0.158
	22		0.103	0.121	0.131	0.140	0.157	0.163
	23		0.107	0.126	0.136	0.145	0.162	0.169
	24		0.112	0.131	0.141	0.150	0.168	0.174
	25		0.116	0.135	0.146	0.155	0.173	0.180
	26		0.121	0.140	0.151	0.160	0.178	0.185
	27		0.126	0.145	0.156	0.165	0.183	0.190

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	28		0.130	0.150	0.161	0.170	0.189	0.196
	29		0.135	0.155	0.166	0.175	0.194	0.201
	30		0.139	0.160	0.171	0.180	0.199	0.206
	31		0.144	0.164	0.176	0.185	0.204	0.211
	32		0.148	0.169	0.181	0.190	0.209	0.217
	33		0.153	0.174	0.186	0.195	0.215	0.222
	34		0.157	0.179	0.190	0.200	0.220	0.227
	35		0.162	0.183	0.195	0.205	0.225	0.232
	36		0.166	0.188	0.200	0.210	0.230	0.237
	37		0.171	0.193	0.205	0.215	0.235	0.242
	38		0.175	0.198	0.210	0.220	0.240	0.248
	39		0.180	0.202	0.215	0.225	0.245	0.253
	40		0.185	0.207	0.220	0.230	0.250	0.258
	41		0.189	0.212	0.224	0.235	0.255	0.263
	42		0.194	0.217	0.229	0.240	0.260	0.268
	43		0.198	0.221	0.234	0.245	0.265	0.273
	44		0.203	0.226	0.239	0.250	0.270	0.278
	45		0.207	0.231	0.244	0.254	0.275	0.283
	46		0.212	0.236	0.248	0.259	0.280	0.288
	47		0.216	0.240	0.253	0.264	0.285	0.293
	48		0.221	0.245	0.258	0.269	0.290	0.298
	49		0.225	0.250	0.263	0.274	0.295	0.303
	50		0.230	0.254	0.268	0.279	0.300	0.308
230	0		0.003	0.007	0.010	0.013	0.020	0.023
	1		0.007	0.013	0.017	0.020	0.029	0.032
	2		0.012	0.019	0.023	0.027	0.036	0.040
	3		0.016	0.024	0.029	0.033	0.043	0.047
	4		0.020	0.029	0.034	0.039	0.050	0.054
	5		0.025	0.034	0.040	0.045	0.056	0.060
	6		0.029	0.039	0.045	0.051	0.062	0.067
	7		0.033	0.044	0.051	0.056	0.068	0.073
	8		0.038	0.049	0.056	0.062	0.074	0.079
	9		0.042	0.054	0.061	0.067	0.080	0.085
	10		0.046	0.059	0.066	0.073	0.086	0.091
	11		0.051	0.064	0.071	0.078	0.091	0.097
	12		0.055	0.069	0.076	0.083	0.097	0.102
	13		0.059	0.073	0.081	0.088	0.102	0.108
	14		0.064	0.078	0.086	0.094	0.108	0.113
	15		0.068	0.083	0.091	0.099	0.113	0.119
	16		0.072	0.088	0.096	0.104	0.119	0.124
	17		0.077	0.092	0.101	0.109	0.124	0.130
	18		0.081	0.097	0.106	0.114	0.129	0.135

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	19		0.085	0.102	0.111	0.119	0.135	0.141
	20		0.090	0.106	0.116	0.124	0.140	0.146
	21		0.094	0.111	0.121	0.129	0.145	0.151
	22		0.098	0.116	0.125	0.134	0.150	0.156
	23		0.103	0.120	0.130	0.139	0.155	0.162
	24		0.107	0.125	0.135	0.144	0.160	0.167
	25		0.111	0.130	0.140	0.145	0.166	0.172
	26		0.116	0.134	0.145	0.153	0.171	0.177
	27		0.120	0.139	0.149	0.158	0.176	0.182
	28		0.124	0.144	0.154	0.163	0.181	0.187
	29		0.129	0.148	0.159	0.168	0.186	0.192
	30		0.133	0.153	0.164	0.173	0.191	0.198
	31		0.137	0.157	0.168	0.178	0.196	0.203
	32		0.142	0.162	0.173	0.182	0.201	0.208
	33		0.146	0.166	0.178	0.187	0.206	0.213
	34		0.151	0.171	0.182	0.192	0.211	0.218
	35		0.155	0.176	0.187	0.197	0.215	0.223
	36		0.159	0.180	0.192	0.201	0.220	0.227
	37		0.164	0.185	0.196	0.206	0.225	0.232
	38		0.168	0.189	0.201	0.211	0.230	0.237
	39		0.172	0.194	0.206	0.216	0.235	0.242
	40		0.177	0.198	0.210	0.220	0.240	0.247
	41		0.181	0.203	0.215	0.225	0.245	0.252
	42		0.185	0.207	0.219	0.230	0.249	0.257
	43		0.190	0.212	0.224	0.234	0.254	0.262
	44		0.194	0.216	0.229	0.239	0.259	0.266
	45		0.198	0.221	0.233	0.244	0.264	0.271
	46		0.203	0.225	0.238	0.248	0.269	0.276
	47		0.207	0.230	0.242	0.253	0.273	0.281
	48		0.211	0.234	0.247	0.258	0.278	0.286
	49		0.216	0.239	0.252	0.262	0.283	0.290
	50		0.220	0.243	0.256	0.267	0.287	0.295
240	0		0.003	0.007	0.010	0.012	0.019	0.022
	1		0.007	0.012	0.016	0.020	0.027	0.031
	2		0.011	0.018	0.022	0.026	0.035	0.038
	3		0.015	0.023	0.028	0.032	0.041	0.045
	4		0.019	0.028	0.033	0.038	0.048	0.052
	5		0.024	0.033	0.038	0.043	0.054	0.058
	6		0.028	0.038	0.043	0.049	0.060	0.064
	7		0.032	0.042	0.049	0.054	0.065	0.070
	8		0.036	0.047	0.054	0.059	0.071	0.076
	9		0.040	0.052	0.059	0.065	0.077	0.081
	10		0.044	0.056	0.063	0.070	0.082	0.087

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.049	0.061	0.068	0.075	0.088	0.093
	12		0.053	0.066	0.073	0.080	0.093	0.098
	13		0.057	0.070	0.078	0.085	0.098	0.103
	14		0.061	0.075	0.083	0.090	0.104	0.109
	15		0.065	0.079	0.088	0.095	0.109	0.114
	16		0.069	0.084	0.092	0.099	0.114	0.119
	17		0.074	0.088	0.097	0.104	0.119	0.125
	18		0.078	0.093	0.102	0.109	0.124	0.130
	19		0.082	0.098	0.106	0.114	0.129	0.135
	20		0.086	0.102	0.111	0.119	0.134	0.140
	21		0.090	0.106	0.116	0.124	0.139	0.145
	22		0.094	0.111	0.120	0.128	0.144	0.150
	23		0.098	0.115	0.125	0.133	0.149	0.155
	24		0.103	0.120	0.129	0.138	0.154	0.160
	25		0.107	0.124	0.134	0.142	0.159	0.165
	26		0.111	0.129	0.139	0.147	0.164	0.170
	27		0.115	0.133	0.143	0.152	0.169	0.175
	28		0.119	0.138	0.148	0.156	0.173	0.180
	29		0.123	0.142	0.152	0.161	0.178	0.185
	30		0.128	0.146	0.157	0.167	0.183	0.190
	31		0.132	0.150	0.161	0.170	0.188	0.194
	32		0.136	0.155	0.166	0.175	0.193	0.199
	33		0.140	0.160	0.170	0.180	0.197	0.204
	34		0.144	0.164	0.175	0.184	0.202	0.209
	35		0.148	0.168	0.179	0.189	0.207	0.214
	36		0.153	0.173	0.184	0.193	0.211	0.218
	37		0.157	0.177	0.188	0.198	0.216	0.223
	38		0.161	0.181	0.193	0.202	0.221	0.228
	39		0.165	0.186	0.197	0.207	0.226	0.233
	40		0.169	0.190	0.202	0.211	0.230	0.237
	41		0.173	0.195	0.206	0.216	0.235	0.242
	42		0.178	0.199	0.211	0.220	0.239	0.247
	43		0.182	0.203	0.215	0.225	0.244	0.251
	44		0.186	0.208	0.219	0.229	0.249	0.256
	45		0.190	0.212	0.224	0.234	0.253	0.260
	46		0.194	0.216	0.228	0.238	0.258	0.265
	47		0.198	0.221	0.233	0.243	0.262	0.270
	48		0.203	0.225	0.237	0.247	0.267	0.274
	49		0.207	0.229	0.241	0.252	0.271	0.279
	50		0.211	0.234	0.246	0.256	0.276	0.283
250	0		0.003	0.006	0.009	0.012	0.018	0.021
	1		0.007	0.012	0.015	0.019	0.026	0.029
	2		0.011	0.017	0.021	0.025	0.033	0.037
	3		0.015	0.022	0.027	0.031	0.040	0.043
	4		0.019	0.027	0.032	0.036	0.046	0.050

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	5		0.023	0.031	0.037	0.042	0.052	0.056
	6		0.027	0.036	0.042	0.047	0.057	0.061
	7		0.031	0.041	0.047	0.052	0.063	0.067
	8		0.035	0.045	0.051	0.057	0.068	0.073
	9		0.039	0.050	0.056	0.062	0.074	0.078
	10		0.043	0.054	0.061	0.067	0.079	0.084
	11		0.047	0.059	0.066	0.072	0.084	0.089
	12		0.051	0.063	0.070	0.077	0.089	0.094
	13		0.055	0.068	0.075	0.081	0.094	0.099
	14		0.059	0.072	0.080	0.086	0.099	0.105
	15		0.063	0.076	0.084	0.091	0.104	0.110
	16		0.067	0.081	0.089	0.096	0.109	0.115
	17		0.071	0.085	0.093	0.100	0.114	0.120
	18		0.075	0.089	0.098	0.105	0.119	0.125
	19		0.079	0.094	0.102	0.110	0.124	0.130
	20		0.083	0.098	0.107	0.114	0.129	0.135
	21		0.087	0.102	0.111	0.119	0.134	0.135
	22		0.091	0.107	0.116	0.123	0.139	0.144
	23		0.095	0.111	0.120	0.128	0.143	0.149
	24		0.099	0.115	0.124	0.132	0.148	0.154
	25		0.103	0.119	0.129	0.137	0.153	0.159
	26		0.107	0.124	0.133	0.141	0.157	0.163
	27		0.111	0.128	0.138	0.146	0.162	0.168
	28		0.115	0.132	0.142	0.150	0.167	0.173
	29		0.119	0.136	0.146	0.155	0.171	0.178
	30		0.123	0.141	0.151	0.159	0.176	0.182
	31		0.127	0.145	0.155	0.164	0.181	0.187
	32		0.130	0.149	0.159	0.168	0.185	0.192
	33		0.134	0.153	0.164	0.173	0.190	0.196
	34		0.138	0.158	0.168	0.177	0.194	0.201
	35		0.142	0.162	0.172	0.181	0.199	0.205
	36		0.146	0.166	0.177	0.186	0.203	0.210
	37		0.150	0.170	0.181	0.190	0.208	0.215
	38		0.154	0.174	0.18?	0.194	0.212	0.219
	39		0.158	0.179	0.189	0.199	0.217	0.224
	40		0.162	0.183	0.194	0.203	0.221	0.228
	41		0.166	0.187	0.198	0.207	0.226	0.233
	42		0.170	0.191	0.202	0.212	0.230	0.237
	43		0.174	0.195	0.207	0.216	0.235	0.242
	44		0.178	0.199	0.211	0.220	0.239	0.246
	45		0.182	0.204	0.215	0.225	0.239	0.251
	46		0.186	0.208	0.219	0.225	0.244	0.251
	47		0.190	0.212	0.224	0.229	0.248	0.255
	48		0.194	0.216	0.228	0.233	0.252	0.259
	49		0.198	0.220	0.232	0.238	0.257	0.264
	50		0.202	0.224	0.236	0.242	0.261	0.268
						0.246	0.265	0.273

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
300	0		0.002	0.005	0.008	0.010	0.015	0.018
	1		0.006	0.010	0.013	0.016	0.022	0.025
	2		0.009	0.014	0.018	0.021	0.028	0.031
	3		0.012	0.018	0.022	0.026	0.033	0.036
	4		0.016	0.022	0.026	0.030	0.038	0.041
	5		0.019	0.026	0.031	0.035	0.043	0.046
	6		0.022	0.030	0.035	0.039	0.048	0.051
	7		0.026	0.034	0.039	0.043	0.053	0.056
	8		0.029	0.038	0.043	0.048	0.057	0.061
	9		0.032	0.041	0.047	0.052	0.062	0.065
	10		0.036	0.045	0.051	0.056	0.066	0.070
	11		0.039	0.049	0.055	0.060	0.070	0.074
	12		0.042	0.053	0.059	0.064	0.075	0.079
	13		0.046	0.056	0.063	0.068	0.079	0.083
	14		0.049	0.060	0.066	0.072	0.083	0.088
	15		0.052	0.064	0.070	0.076	0.087	0.092
	16		0.055	0.067	0.074	0.080	0.092	0.096
	17		0.059	0.071	0.078	0.084	0.096	0.100
	18		0.062	0.075	0.082	0.088	0.100	0.104
	19		0.065	0.078	0.085	0.092	0.104	0.109
	20		0.069	0.082	0.089	0.095	0.108	0.113
	21		0.072	0.085	0.093	0.099	0.112	0.117
	22		0.075	0.089	0.097	0.103	0.116	0.121
	23		0.079	0.093	0.100	0.107	0.120	0.125
	24		0.082	0.096	0.104	0.111	0.124	0.129
	25		0.085	0.100	0.108	0.114	0.128	0.133
	26		0.089	0.103	0.111	0.118	0.132	0.137
	27		0.092	0.107	0.115	0.122	0.136	0.141
	28		0.095	0.110	0.119	0.126	0.140	0.145
	29		0.099	0.114	0.122	0.129	0.144	0.149
	30		0.102	0.117	0.126	0.133	0.147	0.153
	31		0.105	0.121	0.129	0.137	0.151	0.157
	32		0.109	0.124	0.133	0.141	0.155	0.161
	33		0.112	0.128	0.137	0.144	0.159	0.165
	34		0.115	0.132	0.140	0.148	0.163	0.168
	35		0.119	0.135	0.144	0.152	0.167	0.172
	36		0.122	0.139	0.148	0.155	0.170	0.176
	37		0.125	0.142	0.151	0.159	0.174	0.180
	38		0.129	0.146	0.155	0.163	0.178	0.184
	39		0.132	0.149	0.158	0.166	0.182	0.188
	40		0.135	0.153	0.162	0.170	0.186	0.191
	41		0.139	0.156	0.166	0.174	0.189	0.195
	42		0.142	0.160	0.169	0.177	0.193	0.199

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	43		0.145	0.163	0.173	0.181	0.197	0.203
	44		0.149	0.167	0.176	0.185	0.201	0.207
	45		0.152	0.170	0.180	0.188	0.204	0.210
	46		0.155	0.174	0.183	0.192	0.208	0.214
	47		0.159	0.177	0.187	0.195	0.212	0.218
	48		0.162	0.180	0.190	0.199	0.215	0.222
	49		0.165	0.184	0.194	0.203	0.219	0.225
	50		0.169	0.187	0.198	0.206	0.223	0.229
350	0		0.002	0.005	0.007	0.009	0.013	0.015
	1		0.005	0.009	0.011	0.013	0.019	0.021
	2		0.008	0.012	0.015	0.018	0.024	0.026
	3		0.010	0.016	0.019	0.022	0.028	0.031
	4		0.013	0.019	0.023	0.026	0.033	0.036
	5		0.016	0.022	0.026	0.030	0.037	0.040
	6		0.019	0.026	0.030	0.034	0.041	0.044
	7		0.022	0.029	0.033	0.037	0.045	0.048
	8		0.025	0.032	0.037	0.041	0.049	0.052
	9		0.028	0.036	0.040	0.044	0.053	0.056
	10		0.030	0.039	0.044	0.048	0.057	0.060
	11		0.033	0.042	0.047	0.051	0.060	0.064
	12		0.036	0.045	0.050	0.055	0.064	0.068
	13		0.039	0.048	0.054	0.058	0.068	0.072
	14		0.042	0.051	0.057	0.062	0.072	0.075
	15		0.045	0.055	0.060	0.065	0.075	0.079
	16		0.048	0.058	0.064	0.069	0.079	0.083
	17		0.050	0.061	0.067	0.072	0.083	0.086
	18		0.053	0.064	0.070	0.075	0.086	0.090
	19		0.056	0.067	0.073	0.079	0.089	0.093
	20		0.059	0.070	0.076	0.082	0.093	0.097
	21		0.062	0.073	0.080	0.085	0.096	0.101
	22		0.065	0.076	0.083	0.089	0.100	0.104
	23		0.068	0.079	0.086	0.092	0.103	0.108
	24		0.070	0.082	0.089	0.095	0.107	0.111
	25		0.073	0.086	0.092	0.098	0.110	0.114
	26		0.076	0.089	0.096	0.102	0.113	0.118
	27		0.079	0.092	0.099	0.105	0.117	0.121
	28		0.082	0.095	0.102	0.108	0.120	0.125
	29		0.085	0.098	0.105	0.111	0.124	0.128
	30		0.088	0.101	0.108	0.114	0.127	0.132
	31		0.090	0.104	0.111	0.118	0.130	0.135

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	32		0.093	0.107	0.114	0.121	0.134	0.138
	33		0.096	0.110	0.118	0.124	0.137	0.142
	34		0.099	0.113	0.121	0.127	0.140	0.145
	35		0.102	0.116	0.124	0.130	0.143	0.148
	36		0.105	0.119	0.127	0.134	0.147	0.152
	37		0.108	0.122	0.130	0.137	0.150	0.155
	38		0.110	0.125	0.133	0.140	0.153	0.158
	39		0.113	0.128	0.136	0.143	0.156	0.162
	40		0.116	0.131	0.139	0.146	0.160	0.165
	41		0.119	0.134	0.142	0.149	0.163	0.168
	42		0.122	0.137	0.145	0.152	0.166	0.171
	43		0.125	0.140	0.148	0.156	0.169	0.175
	44		0.128	0.143	0.151	0.159	0.173	0.178
	45		0.130	0.146	0.155	0.162	0.176	0.181
	46		0.133	0.149	0.158	0.165	0.179	0.184
	47		0.136	0.152	0.161	0.168	0.182	0.188
	48		0.139	0.155	0.164	0.171	0.185	0.191
	49		0.142	0.158	0.167	0.174	0.189	0.194
	50		0.145	0.161	0.170	0.177	0.192	0.197
400	0		0.002	0.004	0.006	0.007	0.011	0.013
	1		0.004	0.007	0.010	0.012	0.016	0.018
	2		0.007	0.011	0.013	0.016	0.021	0.023
	3		0.009	0.014	0.017	0.019	0.025	0.027
	4		0.012	0.017	0.020	0.023	0.029	0.031
	5		0.014	0.020	0.023	0.026	0.032	0.035
	6		0.017	0.023	0.026	0.029	0.036	0.039
	7		0.019	0.025	0.029	0.033	0.040	0.042
	8		0.022	0.028	0.032	0.036	0.043	0.046
	9		0.024	0.031	0.035	0.039	0.046	0.049
	10		0.027	0.034	0.038	0.042	0.050	0.053
	11		0.029	0.037	0.041	0.045	0.053	0.056
	12		0.032	0.040	0.044	0.048	0.056	0.059
	13		0.034	0.042	0.047	0.051	0.060	0.063
	14		0.037	0.045	0.050	0.054	0.063	0.066
	15		0.039	0.048	0.053	0.057	0.066	0.069
	16		0.042	0.051	0.056	0.060	0.069	0.072
	17		0.044	0.053	0.059	0.063	0.072	0.076
	18		0.047	0.056	0.061	0.066	0.075	0.079
	19		0.049	0.059	0.064	0.069	0.078	0.082
	20		0.052	0.061	0.067	0.072	0.081	0.085

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	21		0.054	0.064	0.070	0.075	0.084	0.088
	22		0.057	0.067	0.073	0.078	0.087	0.091
	23		0.059	0.070	0.075	0.080	0.091	0.094
	24		0.062	0.072	0.078	0.083	0.094	0.097
	25		0.064	0.075	0.081	0.086	0.097	0.100
	26		0.067	0.078	0.084	0.089	0.099	0.103
	27		0.069	0.080	0.087	0.092	0.102	0.106
	28		0.072	0.083	0.089	0.095	0.105	0.110
	29		0.074	0.086	0.092	0.098	0.108	0.112
	30		0.077	0.088	0.095	0.100	0.111	0.115
	31		0.079	0.091	0.098	0.103	0.114	0.118
	32		0.082	0.094	0.100	0.106	0.117	0.121
	33		0.084	0.096	0.103	0.109	0.120	0.124
	34		0.087	0.099	0.106	0.112	0.123	0.127
	35		0.089	0.102	0.108	0.114	0.126	0.130
	36		0.092	0.104	0.111	0.117	0.129	0.133
	37		0.094	0.107	0.114	0.120	0.132	0.136
	38		0.097	0.109	0.117	0.123	0.135	0.139
	39		0.099	0.112	0.119	0.125	0.137	0.142
	40		0.102	0.115	0.122	0.128	0.140	0.145
	41		0.104	0.117	0.125	0.131	0.143	0.148
	42		0.107	0.120	0.127	0.134	0.146	0.150
	43		0.109	0.123	0.130	0.136	0.149	0.153
	44		0.112	0.125	0.133	0.139	0.152	0.156
	45		0.114	0.128	0.135	0.142	0.154	0.159
	46		0.117	0.131	0.138	0.145	0.157	0.162
	47		0.119	0.133	0.141	0.147	0.160	0.165
	48		0.122	0.136	0.144	0.150	0.163	0.168
	49		0.124	0.138	0.146	0.153	0.166	0.171
	50		0.127	0.141	0.149	0.156	0.168	0.173
450	0		0.002	0.004	0.005	0.007	0.010	0.012
	1		0.004	0.007	0.009	0.010	0.015	0.016
	2		0.006	0.009	0.012	0.014	0.019	0.020
	3		0.008	0.012	0.015	0.017	0.022	0.024
	4		0.010	0.015	0.018	0.020	0.026	0.028
	5		0.013	0.018	0.021	0.023	0.029	0.031
	6		0.015	0.020	0.023	0.026	0.032	0.034
	7		0.017	0.023	0.026	0.029	0.035	0.038
	8		0.019	0.025	0.029	0.032	0.038	0.041
	9		0.021	0.028	0.031	0.035	0.041	0.044
	10		0.024	0.030	0.034	0.037	0.044	0.047

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	11		0.026	0.033	0.037	0.040	0.047	0.050
	12		0.028	0.035	0.039	0.043	0.050	0.053
	13		0.030	0.038	0.042	0.046	0.053	0.056
	14		0.033	0.040	0.044	0.048	0.056	0.059
	15		0.035	0.043	0.047	0.051	0.059	0.062
	16		0.037	0.045	0.050	0.054	0.061	0.065
	17		0.039	0.047	0.052	0.056	0.064	0.067
	18		0.041	0.050	0.055	0.059	0.067	0.070
	19		0.044	0.052	0.057	0.061	0.070	0.073
	20		0.046	0.055	0.060	0.064	0.072	0.076
	21		0.048	0.057	0.062	0.067	0.075	0.079
	22		0.050	0.059	0.065	0.069	0.078	0.081
	23		0.053	0.062	0.067	0.072	0.081	0.084
	24		0.055	0.064	0.070	0.074	0.083	0.087
	25		0.057	0.067	0.072	0.077	0.086	0.089
	26		0.059	0.069	0.075	0.079	0.089	0.092
	27		0.061	0.071	0.077	0.082	0.091	0.095
	28		0.064	0.074	0.079	0.084	0.094	0.098
	29		0.066	0.076	0.082	0.087	0.097	0.100
	30		0.068	0.079	0.084	0.089	0.099	0.103
	31		0.070	0.081	0.087	0.092	0.102	0.106
	32		0.073	0.083	0.089	0.094	0.104	0.108
	33		0.075	0.086	0.092	0.097	0.107	0.111
	34		0.077	0.088	0.094	0.099	0.110	0.113
	35		0.079	0.090	0.097	0.102	0.112	0.116
	36		0.081	0.093	0.099	0.104	0.115	0.119
	37		0.084	0.095	0.101	0.107	0.117	0.121
	38		0.086	0.097	0.104	0.109	0.120	0.124
	39		0.088	0.100	0.106	0.112	0.122	0.126
	40		0.090	0.102	0.109	0.114	0.125	0.129
	41		0.093	0.104	0.111	0.117	0.128	0.132
	42		0.095	0.107	0.113	0.119	0.130	0.134
	43		0.097	0.109	0.116	0.121	0.133	0.137
	44		0.099	0.111	0.118	0.124	0.135	0.139
	45		0.101	0.114	0.121	0.126	0.138	0.142
	46		0.104	0.116	0.123	0.129	0.140	0.144
	47		0.106	0.118	0.125	0.131	0.143	0.147
	48		0.108	0.121	0.128	0.134	0.145	0.150
	49		0.110	0.123	0.130	0.136	0.148	0.152
	50		0.113	0.125	0.133	0.139	0.150	0.155
500	0		0.001	0.003	0.005	0.006	0.009	0.011
	1		0.003	0.006	0.008	0.009	0.013	0.015
	2		0.005	0.009	0.011	0.013	0.017	0.018

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	3		0.007	0.011	0.013	0.015	0.020	0.022
	4		0.009	0.013	0.016	0.018	0.023	0.025
	5		0.011	0.016	0.018	0.021	0.026	0.028
	6		0.013	0.018	0.121	0.024	0.029	0.031
	7		0.015	0.020	0.023	0.026	0.032	0.034
	8		0.017	0.023	0.026	0.029	0.034	0.037
	9		0.019	0.025	0.028	0.031	0.037	0.040
	10		0.021	0.027	0.031	0.034	0.040	0.042
	11		0.023	0.029	0.033	0.036	0.043	0.045
	12		0.025	0.032	0.035	0.039	0.045	0.048
	13		0.027	0.034	0.038	0.041	0.048	0.050
	14		0.029	0.036	0.040	0.043	0.050	0.053
	15		0.031	0.038	0.042	0.046	0.053	0.056
	16		0.033	0.040	0.045	0.048	0.055	0.058
	17		0.035	0.043	0.047	0.051	0.058	0.061
	18		0.037	0.045	0.049	0.053	0.060	0.063
	19		0.039	0.047	0.051	0.055	0.063	0.066
	20		0.041	0.049	0.054	0.058	0.065	0.068
	21		0.043	0.051	0.056	0.060	0.068	0.071
	22		0.045	0.054	0.058	0.062	0.070	0.073
	23		0.047	0.056	0.060	0.065	0.073	0.076
	24		0.049	0.058	0.063	0.067	0.075	0.078
	25		0.051	0.060	0.065	0.069	0.077	0.081
	26		0.053	0.062	0.067	0.071	0.080	0.083
	27		0.055	0.064	0.069	0.074	0.082	0.086
	28		0.057	0.066	0.072	0.076	0.085	0.088
	29		0.059	0.069	0.074	0.078	0.087	0.090
	30		0.061	0.071	0.076	0.081	0.089	0.093
	31		0.063	0.073	0.078	0.083	0.092	0.095
	32		0.065	0.075	0.080	0.085	0.094	0.098
	33		0.067	0.077	0.083	0.087	0.096	0.100
	34		0.069	0.079	0.085	0.090	0.099	0.102
	35		0.071	0.081	0.087	0.092	0.101	0.105
	36		0.073	0.083	0.089	0.094	0.103	0.107
	37		0.075	0.086	0.091	0.096	0.106	0.109
	38		0.077	0.088	0.093	0.098	0.108	0.112
	39		0.079	0.090	0.096	0.101	0.110	0.114
	40		0.081	0.092	0.098	0.103	0.113	0.116
	41		0.083	0.094	0.100	0.105	0.115	0.119
	42		0.085	0.096	0.102	0.107	0.117	0.121
	43		0.087	0.098	0.104	0.110	0.120	0.123
	44		0.089	0.100	0.107	0.112	0.122	0.126
	45		0.091	0.102	0.109	0.114	0.124	0.128

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	46		0.093	0.105	0.111	0.116	0.126	0.130
	47		0.095	0.107	0.113	0.118	0.129	0.133
	48		0.097	0.109	0.115	0.120	0.131	0.135
	49		0.099	0.111	0.117	0.123	0.133	0.137
	50		0.101	0.113	0.119	0.125	0.135	0.139
	51		0.103	0.115	0.122	0.127	0.138	0.142
	52		0.105	0.117	0.124	0.129	0.140	0.144
	53		0.107	0.119	0.126	0.131	0.142	0.146
	54		0.109	0.121	0.128	0.134	0.144	0.149
	55		0.111	0.123	0.130	0.136	0.147	0.151
	56		0.113	0.126	0.132	0.138	0.149	0.153
	57		0.115	0.128	0.134	0.140	0.151	0.155
	58		0.117	0.130	0.137	0.142	0.153	0.158
	59		0.119	0.132	0.139	0.144	0.156	0.160
	60		0.121	0.134	0.141	0.147	0.158	0.162
	61		0.123	0.136	0.143	0.149	0.160	0.164
	62		0.125	0.138	0.145	0.151	0.162	0.167
	63		0.127	0.140	0.147	0.153	0.165	0.169
	64		0.129	0.142	0.149	0.155	0.167	0.171
	65		0.131	0.144	0.151	0.157	0.169	0.173
	66		0.133	0.146	0.154	0.160	0.171	0.176
	67		0.135	0.148	0.156	0.162	0.173	0.178
	68		0.137	0.151	0.158	0.164	0.176	0.180
	69		0.139	0.153	0.160	0.166	0.178	0.182
	70		0.141	0.155	0.162	0.168	0.180	0.184
	71		0.143	0.157	0.164	0.170	0.182	0.187
	72		0.145	0.159	0.166	0.172	0.184	0.189
	73		0.147	0.161	0.168	0.175	0.187	0.191
	74		0.149	0.163	0.170	0.177	0.188	0.193
	75		0.151	0.165	0.173	0.179	0.191	0.195
	76		0.153	0.167	0.175	0.181	0.193	0.198
	77		0.155	0.169	0.177	0.183	0.195	0.200
	78		0.157	0.171	0.179	0.185	0.197	0.202
	79		0.159	0.173	0.181	0.187	0.200	0.204
	80		0.161	0.175	0.183	0.189	0.202	0.206
	81		0.163	0.177	0.185	0.192	0.204	0.209
	82		0.165	0.180	0.187	0.194	0.206	0.211
	83		0.167	0.182	0.189	0.196	0.208	0.213
	84		0.169	0.184	0.191	0.198	0.211	0.215
	85		0.171	0.186	0.194	0.200	0.213	0.217
	86		0.173	0.188	0.196	0.202	0.215	0.220
	87		0.175	0.190	0.198	0.204	0.217	0.222
	88		0.177	0.192	0.200	0.206	0.219	0.224
	89		0.179	0.194	0.202	0.209	0.221	0.226

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	90		0.181	0.196	0.204	0.211	0.223	0.228
	91		0.183	0.198	0.206	0.213	0.226	0.230
	92		0.185	0.200	0.208	0.215	0.228	0.233
	93		0.187	0.202	0.210	0.217	0.230	0.235
	94		0.189	0.204	0.212	0.219	0.232	0.237
	95		0.191	0.206	0.214	0.221	0.234	0.239
	96		0.193	0.208	0.216	0.223	0.236	0.241
	97		0.195	0.210	0.219	0.225	0.239	0.243
550	0		0.001	0.003	0.004	0.005	0.008	0.010
	1		0.003	0.005	0.007	0.009	0.012	0.013
	2		0.005	0.008	0.010	0.011	0.015	0.017
	3		0.007	0.010	0.012	0.014	0.018	0.020
	4		0.008	0.012	0.014	0.017	0.021	0.023
	5		0.010	0.014	0.017	0.019	0.024	0.026
	6		0.012	0.016	0.019	0.021	0.026	0.028
	7		0.014	0.019	0.021	0.024	0.029	0.031
	8		0.016	0.021	0.024	0.026	0.031	0.033
	9		0.018	0.023	0.026	0.028	0.034	0.036
	10		0.019	0.025	0.028	0.031	0.036	0.039
	11		0.021	0.027	0.030	0.033	0.039	0.041
	12		0.023	0.029	0.032	0.035	0.041	0.043
	13		0.025	0.031	0.034	0.037	0.043	0.046
	14		0.027	0.033	0.036	0.040	0.046	0.048
	15		0.028	0.035	0.038	0.042	0.048	0.051
	16		0.030	0.037	0.041	0.044	0.050	0.053
	17		0.032	0.039	0.043	0.046	0.053	0.055
	18		0.034	0.041	0.045	0.048	0.055	0.058
	19		0.036	0.043	0.047	0.050	0.057	0.060
	20		0.038	0.045	0.049	0.052	0.059	0.062
	21		0.039	0.047	0.051	0.055	0.062	0.064
	22		0.041	0.049	0.053	0.057	0.064	0.067
	23		0.043	0.051	0.055	0.059	0.066	0.069
	24		0.045	0.053	0.057	0.061	0.068	0.071
	25		0.047	0.055	0.059	0.063	0.071	0.073
	26		0.048	0.057	0.061	0.065	0.073	0.076
	27		0.050	0.058	0.063	0.067	0.075	0.078
	28		0.052	0.060	0.065	0.069	0.077	0.080
	29		0.054	0.062	0.067	0.071	0.079	0.082
	30		0.056	0.064	0.069	0.073	0.081	0.084
	31		0.058	0.066	0.071	0.075	0.084	0.087
	32		0.059	0.068	0.073	0.077	0.086	0.089
	33		0.061	0.070	0.075	0.079	0.088	0.091

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	34		0.063	0.072	0.077	0.081	0.090	0.093
	35		0.065	0.074	0.079	0.083	0.092	0.095
	36		0.067	0.076	0.081	0.086	0.094	0.097
	37		0.068	0.078	0.083	0.088	0.096	0.100
	38		0.070	0.080	0.085	0.090	0.098	0.102
	39		0.072	0.082	0.087	0.092	0.101	0.104
	40		0.074	0.084	0.089	0.094	0.103	0.106
	41		0.076	0.086	0.091	0.096	0.105	0.108
	42		0.078	0.087	0.093	0.098	0.107	0.110
	43		0.079	0.089	0.095	0.100	0.109	0.112
	44		0.081	0.091	0.097	0.102	0.111	0.114
	45		0.083	0.093	0.099	0.104	0.113	0.117
	46		0.085	0.095	0.101	0.106	0.115	0.119
	47		0.087	0.097	0.103	0.108	0.117	0.121
	48		0.088	0.099	0.105	0.110	0.119	0.123
	49		0.090	0.101	0.107	0.112	0.121	0.125
	50		0.092	0.103	0.109	0.114	0.123	0.127
	51		0.093	0.105	0.111	0.116	0.125	0.129
	52		0.096	0.107	0.113	0.118	0.127	0.131
	53		0.098	0.109	0.115	0.120	0.130	0.133
	54		0.099	0.110	0.116	0.122	0.132	0.135
	55		0.101	0.112	0.118	0.124	0.134	0.137
	56		0.103	0.114	0.120	0.126	0.136	0.139
	57		0.105	0.116	0.122	0.128	0.138	0.142
	58		0.107	0.118	0.124	0.130	0.140	0.144
	59		0.108	0.120	0.126	0.131	0.142	0.146
	60		0.110	0.122	0.128	0.133	0.144	0.148
	61		0.112	0.124	0.130	0.135	0.146	0.150
	62		0.114	0.126	0.132	0.137	0.148	0.152
	63		0.116	0.127	0.134	0.139	0.150	0.154
	64		0.118	0.129	0.136	0.141	0.152	0.156
	65		0.119	0.131	0.138	0.143	0.154	0.158
	66		0.121	0.133	0.140	0.145	0.156	0.160
	67		0.123	0.135	0.142	0.147	0.158	0.162
	68		0.125	0.137	0.144	0.149	0.160	0.164
	69		0.127	0.139	0.145	0.151	0.162	0.166
	70		0.128	0.141	0.147	0.153	0.164	0.168
	71		0.130	0.143	0.149	0.155	0.166	0.170
	72		0.132	0.145	0.151	0.157	0.168	0.172
	73		0.134	0.146	0.153	0.159	0.170	0.174
	74		0.136	0.148	0.155	0.161	0.172	0.176
	75		0.137	0.150	0.157	0.161	0.174	0.178
	76		0.139	0.152	0.159	0.165	0.176	0.180
	77		0.141	0.154	0.161	0.167	0.178	0.182

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
600	0		0.001	0.003	0.004	0.005	0.008	0.009
	1		0.003	0.005	0.006	0.008	0.011	0.012
	2		0.004	0.007	0.009	0.010	0.014	0.015
	3		0.006	0.009	0.011	0.013	0.017	0.018
	4		0.008	0.011	0.013	0.015	0.019	0.021
	5		0.009	0.013	0.015	0.017	0.022	0.023
	6		0.011	0.015	0.017	0.020	0.024	0.026
	7		0.013	0.017	0.020	0.022	0.026	0.028
	8		0.014	0.019	0.022	0.024	0.029	0.031
	9		0.016	0.021	0.024	0.026	0.031	0.033
	10		0.018	0.023	0.026	0.028	0.033	0.035
	11		0.019	0.025	0.028	0.030	0.036	0.038
	12		0.021	0.026	0.029	0.032	0.038	0.040
	13		0.023	0.028	0.031	0.034	0.040	0.042
	14		0.024	0.030	0.033	0.036	0.042	0.044
	15		0.026	0.032	0.035	0.038	0.044	0.046
	16		0.028	0.034	0.037	0.040	0.046	0.049
	17		0.029	0.036	0.039	0.042	0.048	0.051
	18		0.031	0.037	0.041	0.044	0.050	0.053
	19		0.033	0.039	0.043	0.046	0.053	0.055
	20		0.034	0.041	0.045	0.048	0.055	0.057
	21		0.036	0.043	0.047	0.050	0.057	0.059
	22		0.038	0.045	0.049	0.052	0.059	0.061
	23		0.039	0.046	0.050	0.054	0.061	0.063
	24		0.041	0.048	0.052	0.056	0.063	0.065
	25		0.043	0.050	0.054	0.058	0.065	0.067
	26		0.044	0.052	0.056	0.060	0.067	0.069
	27		0.046	0.054	0.058	0.062	0.069	0.071
	28		0.048	0.055	0.060	0.063	0.071	0.074
	29		0.049	0.057	0.062	0.065	0.073	0.076
	30		0.051	0.059	0.063	0.067	0.075	0.078
	31		0.053	0.061	0.065	0.069	0.077	0.080
	32		0.054	0.063	0.067	0.071	0.079	0.082
	33		0.056	0.064	0.069	0.073	0.081	0.084
	34		0.058	0.066	0.071	0.075	0.083	0.086
	35		0.059	0.068	0.073	0.077	0.085	0.088
	36		0.061	0.070	0.074	0.078	0.086	0.089
	37		0.063	0.071	0.076	0.080	0.088	0.091
	38		0.064	0.073	0.078	0.082	0.090	0.093
	39		0.066	0.075	0.080	0.084	0.092	0.095
	40		0.068	0.077	0.082	0.086	0.094	0.097
	41		0.069	0.078	0.083	0.088	0.096	0.099

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	42		0.071	0.080	0.085	0.090	0.098	0.101
	43		0.073	0.082	0.087	0.091	0.100	0.103
	44		0.074	0.084	0.089	0.093	0.102	0.105
	45		0.076	0.086	0.091	0.095	0.104	0.107
	46		0.078	0.087	0.093	0.097	0.106	0.109
	47		0.079	0.089	0.094	0.099	0.108	0.111
	48		0.081	0.091	0.096	0.101	0.109	0.113
	49		0.083	0.093	0.098	0.102	0.111	0.115
	50		0.084	0.094	0.100	0.104	0.113	0.117
	51		0.086	0.096	0.101	0.106	0.115	0.119
	52		0.088	0.098	0.103	0.108	0.117	0.120
	53		0.089	0.100	0.105	0.110	0.119	0.122
	54		0.091	0.101	0.107	0.112	0.121	0.124
	55		0.093	0.103	0.109	0.113	0.123	0.126
	56		0.094	0.105	0.110	0.115	0.125	0.128
	57		0.096	0.106	0.112	0.117	0.126	0.130
	58		0.098	0.108	0.114	0.119	0.128	0.132
	59		0.099	0.110	0.116	0.121	0.130	0.134
	60		0.101	0.112	0.118	0.122	0.132	0.136
	61		0.103	0.113	0.119	0.124	0.134	0.138
	62		0.104	0.115	0.121	0.126	0.136	0.139
	63		0.106	0.117	0.123	0.128	0.138	0.141
	64		0.108	0.119	0.125	0.130	0.139	0.143
	65		0.109	0.120	0.126	0.131	0.141	0.145
	66		0.111	0.122	0.128	0.133	0.143	0.147
	67	0.113	0.113	0.124	0.130	0.135	0.145	0.149
	68		0.114	0.126	0.132	0.137	0.147	0.151
	69		0.116	0.127	0.133	0.138	0.149	0.152
	70		0.118	0.129	0.135	0.140	0.151	0.154
	71		0.119	0.131	0.137	0.142	0.152	0.156
	72		0.121	0.133	0.139	0.144	0.154	0.158
	73		0.123	0.134	0.141	0.146	0.156	0.160
650	0		0.001	0.002	0.004	0.005	0.007	0.008
	1		0.003	0.005	0.006	0.007	0.010	0.011
	2		0.004	0.007	0.008	0.010	0.013	0.014
	3		0.006	0.008	0.010	0.012	0.015	0.017
	4		0.007	0.010	0.012	0.014	0.018	0.019
	5		0.009	0.012	0.014	0.016	0.020	0.022

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	6		0.010	0.014	0.016	0.018	0.022	0.024
	7		0.012	0.016	0.018	0.020	0.024	0.026
	8		0.013	0.017	0.019	0.022	0.027	0.028
	9		0.015	0.019	0.022	0.024	0.029	0.031
	10		0.016	0.021	0.024	0.026	0.031	0.033
	11		0.018	0.023	0.025	0.028	0.033	0.035
	12		0.019	0.024	0.027	0.030	0.035	0.037
	13		0.021	0.026	0.029	0.032	0.037	0.039
	14		0.023	0.028	0.031	0.033	0.039	0.041
	15		0.024	0.029	0.033	0.035	0.041	0.043
	16		0.026	0.031	0.034	0.037	0.043	0.045
	17		0.027	0.033	0.036	0.039	0.045	0.047
	18		0.029	0.035	0.038	0.041	0.047	0.049
	19		0.030	0.036	0.040	0.043	0.049	0.051
	20		0.032	0.038	0.041	0.044	0.050	0.053
	21		0.033	0.040	0.043	0.046	0.052	0.055
	22		0.035	0.041	0.045	0.048	0.054	0.057
	23		0.036	0.043	0.047	0.050	0.056	0.059
	24		0.038	0.045	0.048	0.052	0.058	0.060
	25		0.039	0.046	0.050	0.053	0.060	0.062
	26		0.041	0.048	0.052	0.055	0.062	0.064
	27		0.043	0.050	0.053	0.057	0.064	0.066
	28		0.044	0.051	0.055	0.059	0.065	0.068
	29		0.046	0.053	0.057	0.060	0.067	0.070
	30		0.047	0.054	0.059	0.062	0.069	0.072
	31		0.049	0.056	0.060	0.064	0.071	0.074
	32		0.050	0.058	0.062	0.066	0.073	0.075
	33		0.052	0.059	0.064	0.067	0.074	0.077
	34		0.053	0.061	0.065	0.069	0.076	0.079
	35		0.055	0.063	0.067	0.071	0.078	0.081
	36		0.056	0.064	0.069	0.072	0.080	0.083
	37		0.058	0.066	0.070	0.074	0.082	0.085
	38		0.059	0.068	0.072	0.076	0.083	0.086
	39		0.061	0.069	0.074	0.078	0.085	0.088
	40		0.063	0.071	0.075	0.079	0.087	0.090
	41		0.064	0.072	0.077	0.081	0.089	0.092
	42		0.065	0.074	0.079	0.083	0.091	0.094
	43		0.067	0.076	0.080	0.084	0.092	0.095
	44		0.069	0.077	0.082	0.086	0.094	0.097
	45		0.070	0.079	0.084	0.088	0.096	0.099
	46		0.072	0.081	0.085	0.090	0.098	0.101
	47		0.073	0.082	0.087	0.091	0.099	0.103
	48		0.075	0.084	0.089	0.093	0.101	0.104
	49		0.076	0.085	0.090	0.095	0.103	0.106
	50		0.078	0.087	0.092	0.096	0.105	0.108

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	51		0.079	0.089	0.094	0.098	0.106	0.110
	52		0.081	0.090	0.095	0.100	0.108	0.111
	53		0.083	0.092	0.097	0.101	0.110	0.113
	54		0.084	0.094	0.099	0.103	0.112	0.115
	55		0.086	0.095	0.100	0.105	0.113	0.117
	56		0.087	0.097	0.102	0.106	0.115	0.118
	57		0.089	0.098	0.104	0.108	0.117	0.120
	58		0.090	0.100	0.105	0.110	0.119	0.122
	59		0.092	0.102	0.107	0.111	0.120	0.124
	60		0.093	0.103	0.109	0.113	0.122	0.125
	61		0.095	0.105	0.110	0.115	0.124	0.127
	62		0.096	0.106	0.112	0.117	0.125	0.129
	63		0.097	0.108	0.114	0.118	0.127	0.131
	64		0.099	0.110	0.115	0.120	0.129	0.132
	65		0.101	0.111	0.117	0.122	0.131	0.134
	66		0.103	0.113	0.118	0.123	0.132	0.136
	67		0.104	0.114	0.120	0.125	0.134	0.138
	68		0.106	0.116	0.122	0.126	0.136	0.139
	69		0.107	0.118	0.123	0.128	0.137	0.141
	70		0.109	0.119	0.125	0.130	0.139	0.143
700	0		0.001	0.002	0.003	0.004	0.007	0.008
	1		0.002	0.004	0.006	0.007	0.009	0.011
	2		0.003	0.006	0.008	0.009	0.012	0.013
	3		0.005	0.008	0.010	0.011	0.014	0.016
	4		0.007	0.010	0.011	0.013	0.016	0.018
	5		0.008	0.011	0.013	0.015	0.019	0.020
	6		0.010	0.013	0.015	0.017	0.021	0.022
	7		0.011	0.015	0.017	0.019	0.023	0.024
	8		0.012	0.016	0.018	0.021	0.025	0.026
	9		0.014	0.018	0.020	0.022	0.027	0.028
	10		0.015	0.019	0.022	0.024	0.029	0.030
	11		0.017	0.021	0.024	0.026	0.030	0.032
	12		0.018	0.023	0.025	0.028	0.032	0.034
	13		0.020	0.024	0.027	0.029	0.034	0.036
	14		0.021	0.026	0.029	0.031	0.036	0.038
	15		0.022	0.027	0.030	0.033	0.038	0.040
	16		0.024	0.029	0.032	0.035	0.040	0.042
	17		0.025	0.031	0.034	0.036	0.042	0.044
	18		0.027	0.032	0.035	0.038	0.043	0.045
	19		0.028	0.034	0.037	0.040	0.045	0.047
	20		0.030	0.035	0.038	0.041	0.047	0.049
	21		0.031	0.037	0.040	0.043	0.049	0.05
	22		0.032	0.038	0.042	0.045	0.050	0.
	23		0.034	0.040	0.043	0.046	0.052	0. 4
	24		0.035	0.041	0.045	0.048	0.054	0.056
	25		0.037	0.043	0.046	0.050	0.056	0.058
	26		0.038	0.044	0.048	0.051	0.057	0.060

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	27		0.040	0.046	0.050	0.053	0.059	0.061
	28		0.041	0.048	0.051	0.054	0.061	0.063
	29		0.042	0.049	0.053	0.056	0.062	0.065
	30		0.044	0.051	0.054	0.058	0.064	0.067
	31		0.045	0.052	0.056	0.059	0.066	0.068
	32		0.047	0.054	0.058	0.061	0.068	0.070
	33		0.048	0.055	0.059	0.063	0.069	0.072
	34		0.050	0.057	0.061	0.064	0.071	0.073
	35		0.051	0.058	0.062	0.066	0.073	0.075
	36		0.052	0.060	0.064	0.067	0.074	0.077
	37		0.054	0.061	0.065	0.069	0.076	0.079
	38		0.055	0.063	0.067	0.071	0.078	0.080
	39		0.057	0.064	0.069	0.072	0.079	0.082
	40		0.058	0.066	0.070	0.074	0.081	0.084
	41		0.059	0.067	0.072	0.075	0.083	0.085
	42		0.061	0.069	0.073	0.077	0.084	0.087
	43		0.062	0.070	0.075	0.079	0.086	0.089
	44		0.064	0.072	0.076	0.080	0.088	0.090
	45		0.065	0.073	0.078	0.082	0.089	0.092
	46		0.067	0.075	0.079	0.083	0.091	0.094
	47		0.068	0.076	0.081	0.085	0.092	0.095
	48		0.069	0.078	0.082	0.086	0.094	0.097
	49		0.071	0.079	0.084	0.088	0.096	0.099
	50		0.072	0.081	0.086	0.090	0.097	0.100
	51		0.074	0.082	0.087	0.091	0.099	0.102
	52		0.075	0.084	0.089	0.093	0.101	0.104
	53		0.077	0.085	0.090	0.094	0.102	0.105
	54		0.078	0.087	0.092	0.096	0.104	0.107
	55		0.079	0.088	0.093	0.097	0.105	0.108
	56		0.081	0.090	0.095	0.099	0.107	0.110
	57		0.082	0.091	0.096	0.100	0.109	0.112
	58		0.084	0.093	0.098	0.102	0.110	0.113
	59		0.085	0.094	0.099	0.104	0.112	0.115
	60		0.087	0.096	0.101	0.105	0.113	0.117
	61		0.088	0.097	0.102	0.107	0.115	0.118
	62		0.089	0.099	0.104	0.108	0.117	0.120
	63		0.091	0.100	0.105	0.110	0.118	0.121
	64		0.092	0.102	0.107	0.111	0.120	0.123
	65		0.094	0.103	0.109	0.113	0.121	0.125
	66		0.095	0.105	0.110	0.114	0.123	0.126
	67		0.097	0.106	0.112	0.116	0.125	0.128

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
750	0		0.001	0.002	0.003	0.004	0.006	0.007
	1		0.002	0.004	0.005	0.006	0.009	0.010
	2		0.004	0.006	0.007	0.008	0.011	0.012
	3		0.005	0.007	0.009	0.010	0.013	0.015
	4		0.006	0.009	0.011	0.012	0.015	0.017
	5		0.008	0.011	0.012	0.014	0.017	0.019
	6		0.009	0.012	0.014	0.016	0.019	0.021
	7		0.010	0.014	0.016	0.017	0.021	0.023
	8		0.012	0.015	0.017	0.019	0.023	0.025
	9		0.013	0.017	0.019	0.021	0.025	0.026
	10		0.014	0.018	0.020	0.023	0.027	0.028
	11		0.016	0.020	0.022	0.024	0.028	0.030
	12		0.017	0.021	0.024	0.026	0.030	0.032
	13		0.018	0.023	0.025	0.027	0.032	0.034
	14		0.020	0.024	0.027	0.029	0.034	0.035
	15		0.021	0.026	0.028	0.031	0.035	0.037
	16		0.022	0.027	0.030	0.032	0.037	0.039
	17		0.024	0.029	0.031	0.034	0.039	0.041
	18		0.025	0.030	0.033	0.035	0.040	0.042
	19		0.026	0.031	0.034	0.037	0.042	0.044
	20		0.028	0.033	0.036	0.039	0.044	0.046
	21		0.029	0.034	0.037	0.040	0.045	0.047
	22		0.030	0.036	0.039	0.042	0.047	0.049
	23		0.032	0.037	0.040	0.043	0.049	0.051
	24		0.033	0.039	0.042	0.045	0.050	0.052
	25		0.034	0.040	0.043	0.046	0.052	0.054
	26		0.036	0.042	0.045	0.048	0.054	0.056
	27		0.037	0.043	0.046	0.049	0.055	0.057
	28		0.038	0.044	0.048	0.051	0.057	0.059
	29		0.040	0.046	0.049	0.052	0.058	0.061
	30		0.041	0.047	0.051	0.054	0.060	0.062
	31		0.042	0.049	0.052	0.055	0.061	0.064
	32		0.044	0.050	0.054	0.057	0.063	0.065
	33		0.045	0.052	0.055	0.058	0.065	0.067
	34		0.046	0.053	0.057	0.060	0.066	0.069
	35		0.048	0.054	0.058	0.061	0.068	0.070
	36		0.049	0.056	0.060	0.063	0.069	0.072
	37		0.050	0.057	0.061	0.064	0.071	0.073
	38		0.052	0.059	0.063	0.066	0.072	0.075
	39		0.053	0.060	0.064	0.067	0.074	0.077
	40		0.054	0.061	0.065	0.069	0.076	0.078
	41		0.056	0.063	0.067	0.070	0.077	0.080

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	42		0.057	0.064	0.068	0.072	0.079	0.081
	43		0.058	0.066	0.070	0.073	0.080	0.083
	44		0.060	0.067	0.071	0.075	0.082	0.084
	45		0.061	0.068	0.073	0.076	0.083	0.086
	46		0.062	0.070	0.074	0.078	0.085	0.087
	47		0.064	0.071	0.076	0.079	0.086	0.089
	48		0.065	0.073	0.077	0.081	0.088	0.091
	49		0.066	0.074	0.078	0.082	0.089	0.092
	50		0.068	0.076	0.080	0.084	0.091	0.094
	51		0.069	0.077	0.081	0.085	0.092	0.095
	52		0.070	0.078	0.083	0.087	0.094	0.097
	53		0.072	0.080	0.084	0.088	0.095	0.098
	54		0.073	0.081	0.086	0.089	0.097	0.100
	55		0.074	0.083	0.087	0.091	0.098	0.101
	56		0.076	0.084	0.088	0.092	0.100	0.103
	57		0.077	0.086	0.090	0.094	0.102	0.104
	58		0.078	0.087	0.091	0.095	0.103	0.106
	59		0.080	0.089	0.093	0.097	0.105	0.107
	60		0.081	0.089	0.094	0.098	0.106	0.109
	51		0.082	0.091	0.096	0.100	0.108	0.110
	62		0.084	0.092	0.097	0.101	0.109	0.112
	63		0.085	0.094	0.098	0.103	0.110	0.113
	64		0.086	0.095	0.100	0.104	0.112	0.115
	65		0.088	0.096	0.101	0.105	0.113	0.117
800	0		0.001	0.002	0.003	0.004	0.006	0.007
	1		0.002	0.004	0.005	0.006	0.008	0.009
	2		0.003	0.005	0.007	0.008	0.010	0.012
	3		0.005	0.007	0.008	0.010	0.013	0.014
	4		0.006	0.008	0.010	0.011	0.014	0.016
	5		0.007	0.010	0.012	0.013	0.016	0.018
	6		0.008	0.011	0.013	0.015	0.018	0.019
	7		0.010	0.013	0.015	0.016	0.020	0.021
	8		0.011	0.014	0.016	0.018	0.021	0.023
	9		0.012	0.016	0.018	0.020	0.023	0.025
	10		0.013	0.017	0.019	0.021	0.025	0.027
	11		0.015	0.018	0.021	0.023	0.026	0.028
	12		0.016	0.020	0.022	0.024	0.028	0.030
	13		0.017	0.021	0.024	0.026	0.029	0.032
	14		0.018	0.023	0.025	0.027	0.031	0.033

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	15		0.020	0.024	0.027	0.029	0.033	0.035
	16		0.021	0.025	0.028	0.030	0.034	0.037
	17		0.022	0.027	0.029	0.032	0.036	0.038
	18		0.023	0.028	0.031	0.033	0.037	0.040
	19		0.025	0.029	0.032	0.035	0.039	0.041
	20		0.026	0.031	0.034	0.036	0.041	0.043
	21		0.027	0.032	0.035	0.038	0.042	0.045
	22		0.028	0.034	0.036	0.039	0.044	0.046
	23		0.030	0.035	0.038	0.040	0.045	0.048
	24		0.031	0.036	0.039	0.042	0.047	0.049
	25		0.032	0.038	0.041	0.043	0.048	0.051
	26		0.033	0.039	0.042	0.045	0.050	0.052
	27		0.035	0.040	0.043	0.046	0.051	0.054
	28		0.036	0.042	0.045	0.048	0.053	0.055
	29		0.037	0.043	0.046	0.049	0.054	0.059
	30		0.038	0.044	0.048	0.051	0.056	0.058
	31		0.040	0.046	0.049	0.052	0.057	0.060
	32		0.041	0.047	0.050	0.053	0.059	0.061
	33		0.042	0.048	0.052	0.055	0.060	0.063
	34		0.043	0.050	0.053	0.056	0.062	0.064
	35		0.045	0.051	0.055	0.058	0.063	0.066
	36		0.046	0.052	0.056	0.058	0.065	0.067
	37		0.047	0.054	0.057	0.060	0.066	0.069
	38		0.048	0.055	0.059	0.062	0.067	0.070
	39		0.050	0.056	0.060	0.063	0.069	0.072
	40		0.051	0.058	0.061	0.065	0.070	0.073
	41		0.052	0.059	0.063	0.066	0.072	0.075
	42		0.053	0.060	0.064	0.067	0.073	0.076
	43		0.055	0.062	0.065	0.069	0.075	0.078
	44		0.056	0.063	0.067	0.070	0.076	0.079
	45		0.057	0.064	0.068	0.072	0.078	0.081
	46		0.058	0.066	0.070	0.072	0.079	0.082
	47		0.060	0.067	0.071	0.074	0.081	0.084
	48		0.061	0.068	0.072	0.076	0.082	0.085
	49		0.062	0.069	0.073	0.077	0.083	0.086
	50		0.063	0.071	0.075	0.078	0.085	0.088
	51		0.065	0.072	0.076	0.080	0.087	0.089
	52		0.066	0.073	0.078	0.081	0.088	0.091
	53		0.067	0.075	0.079	0.083	0.090	0.092
	54		0.068	0.076	0.080	0.084	0.091	0.094
	55		0.070	0.077	0.082	0.085	0.092	0.095
	56		0.070	0.079	0.083	0.087	0.094	0.097
	57		0.072	0.080	0.084	0.088	0.095	0.098
	58		0.073	0.081	0.086	0.089	0.097	0.099
	59		0.075	0.083	0.087	0.091	0.098	0.101

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	15		0.020	0.024	0.027	0.029	0.033	0.035
	16		0.021	0.025	0.028	0.030	0.034	0.037
	17		0.022	0.027	0.029	0.032	0.036	0.038
	18		0.023	0.028	0.031	0.033	0.037	0.040
	19		0.025	0.029	0.032	0.035	0.039	0.041
	20		0.026	0.031	0.034	0.036	0.041	0.043
	21		0.027	0.032	0.035	0.038	0.042	0.045
	22		0.028	0.034	0.036	0.039	0.044	0.046
	23		0.030	0.035	0.038	0.040	0.045	0.048
	24		0.031	0.036	0.039	0.042	0.047	0.049
	25		0.032	0.038	0.041	0.043	0.048	0.051
	26		0.033	0.039	0.042	0.045	0.050	0.052
	27		0.035	0.040	0.043	0.046	0.051	0.054
	28		0.036	0.042	0.045	0.048	0.053	0.055
	29		0.037	0.043	0.046	0.049	0.054	0.059
	30		0.038	0.044	0.048	0.051	0.056	0.058
	31		0.040	0.046	0.049	0.052	0.057	0.060
	32		0.041	0.047	0.050	0.053	0.059	0.061
	33		0.042	0.048	0.052	0.055	0.060	0.063
	34		0.043	0.050	0.053	0.056	0.062	0.064
	35		0.045	0.051	0.055	0.058	0.063	0.066
	36		0.046	0.052	0.056	0.058	0.065	0.067
	37		0.047	0.054	0.057	0.060	0.066	0.069
	38		0.048	0.055	0.059	0.062	0.067	0.070
	39		0.050	0.056	0.060	0.063	0.069	0.072
	40		0.051	0.058	0.061	0.065	0.070	0.073
	41		0.052	0.059	0.063	0.066	0.072	0.075
	42		0.053	0.060	0.064	0.067	0.073	0.076
	43		0.055	0.062	0.065	0.069	0.075	0.078
	44		0.056	0.063	0.067	0.070	0.076	0.079
	45		0.057	0.064	0.068	0.072	0.078	0.081
	46		0.058	0.066	0.070	0.072	0.079	0.082
	47		0.060	0.067	0.071	0.074	0.081	0.084
	48		0.061	0.068	0.072	0.076	0.082	0.085
	49		0.062	0.069	0.073	0.077	0.083	0.086
	50		0.063	0.071	0.075	0.078	0.085	0.088
	51		0.065	0.072	0.076	0.080	0.087	0.089
	52		0.066	0.073	0.078	0.081	0.088	0.091
	53		0.067	0.075	0.079	0.083	0.090	0.092
	54		0.068	0.076	0.080	0.084	0.091	0.094
	55		0.070	0.077	0.082	0.085	0.092	0.095
	56		0.070	0.079	0.083	0.087	0.094	0.097
	57		0.072	0.080	0.084	0.088	0.095	0.098
	58		0.073	0.081	0.086	0.089	0.097	0.099
	59		0.075	0.083	0.087	0.091	0.098	0.101

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	42		0.050	0.057	0.060	0.063	0.070	0.072
	43		0.051	0.058	0.062	0.065	0.071	0.073
	44		0.053	0.059	0.063	0.066	0.072	0.075
	45		0.054	0.060	0.064	0.067	0.074	0.076
	46		0.055	0.062	0.065	0.069	0.075	0.077
	47		0.056	0.063	0.067	0.070	0.076	0.079
	48		0.057	0.064	0.068	0.071	0.078	0.080
	49		0.058	0.065	0.069	0.073	0.079	0.081
	50		0.060	0.067	0.071	0.074	0.080	0.083
	51		0.061	0.068	0.072	0.075	0.082	0.084
	52		0.062	0.069	0.073	0.076	0.083	0.086
	53		0.063	0.070	0.074	0.078	0.084	0.087
	54		0.064	0.071	0.076	0.079	0.086	0.088
	55		0.065	0.072	0.077	0.080	0.087	0.090
	56		0.067	0.074	0.078	0.082	0.088	0.091
	57		0.068	0.075	0.079	0.083	0.090	0.092
	58		0.069	0.077	0.081	0.084	0.091	0.094
	59		0.070	0.078	0.082	0.085	0.092	0.095
	60		0.071	0.079	0.083	0.087	0.094	0.096
	61		0.073	0.080	0.084	0.088	0.095	0.098
	62		0.074	0.081	0.086	0.089	0.096	0.099
900	0		0.001	0.002	0.003	0.003	0.005	0.006
	1		0.002	0.003	0.004	0.005	0.007	0.008
	2		0.003	0.004	0.006	0.007	0.009	0.010
	3		0.004	0.006	0.007	0.009	0.011	0.012
	4		0.005	0.007	0.009	0.010	0.013	0.014
	5		0.006	0.009	0.010	0.012	0.014	0.016
	6		0.007	0.010	0.012	0.013	0.016	0.017
	7		0.009	0.011	0.013	0.015	0.018	0.019
	8		0.010	0.013	0.014	0.016	0.019	0.021
	9		0.011	0.014	0.016	0.017	0.021	0.022
	10		0.012	0.015	0.017	0.019	0.022	0.024
	11		0.013	0.016	0.018	0.020	0.024	0.025
	12		0.014	0.018	0.020	0.022	0.025	0.027
	13		0.015	0.019	0.021	0.023	0.027	0.028
	14		0.016	0.020	0.022	0.024	0.028	0.030
	15		0.017	0.021	0.024	0.026	0.030	0.031
	16		0.019	0.023	0.025	0.027	0.031	0.033
	17		0.020	0.024	0.026	0.028	0.032	0.034
	18		0.021	0.025	0.027	0.030	0.034	0.035
	19		0.022	0.026	0.029	0.031	0.035	0.037

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	20		0.023	0.027	0.030	0.032	0.037	0.038
	21		0.024	0.029	0.031	0.033	0.038	0.040
	22		0.025	0.030	0.032	0.035	0.039	0.041
	23		0.026	0.031	0.034	0.036	0.041	0.042
	24		0.027	0.032	0.034	0.037	0.042	0.043
	25		0.029	0.033	0.036	0.039	0.043	0.045
	26		0.030	0.035	0.037	0.040	0.045	0.047
	27		0.031	0.036	0.039	0.041	0.046	0.048
	28		0.032	0.037	0.040	0.042	0.047	0.049
	29		0.033	0.038	0.041	0.044	0.049	0.051
	30		0.034	0.039	0.042	0.045	0.050	0.052
	31		0.035	0.041	0.044	0.046	0.051	0.053
	32		0.036	0.042	0.045	0.047	0.053	0.055
	33		0.037	0.043	0.046	0.049	0.054	0.056
	34		0.039	0.044	0.047	0.050	0.055	0.057
	35		0.040	0.045	0.049	0.051	0.057	0.059
	36		0.041	0.047	0.050	0.052	0.058	0.060
	37		0.042	0.048	0.051	0.054	0.059	0.061
	38		0.043	0.049	0.052	0.055	0.061	0.063
	39		0.044	0.050	0.053	0.056	0.062	0.064
	40		0.045	0.051	0.055	0.057	0.063	0.065
	41		0.046	0.052	0.056	0.059	0.064	0.067
	42		0.047	0.054	0.057	0.060	0.066	0.068
	43		0.049	0.055	0.058	0.061	0.067	0.069
	44		0.050	0.056	0.059	0.062	0.068	0.070
	45		0.051	0.057	0.061	0.064	0.070	0.072
	46		0.052	0.058	0.062	0.065	0.071	0.073
	47		0.053	0.059	0.063	0.066	0.072	0.074
	48		0.054	0.061	0.064	0.067	0.073	0.076
	49		0.055	0.062	0.065	0.069	0.075	0.077
	50		0.056	0.063	0.067	0.070	0.076	0.078
	51		0.057	0.064	0.068	0.071	0.077	0.080
	52		0.058	0.065	0.069	0.072	0.078	0.081
	53		0.060	0.066	0.070	0.073	0.080	0.082
	54		0.061	0.068	0.071	0.075	0.081	0.083
	55		0.062	0.069	0.073	0.076	0.082	0.085
	56		0.063	0.070	0.074	0.077	0.084	0.086
	57		0.064	0.071	0.075	0.078	0.085	0.087
	58		0.065	0.072	0.076	0.080	0.086	0.088
	59		0.066	0.073	0.077	0.081	0.087	0.090
	60		0.067	0.075	0.079	0.082	0.089	0.091

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
950	0		0.001	0.002	0.002	0.003	0.005	0.006
	1		0.002	0.003	0.004	0.005	0.007	0.008
	2		0.003	0.004	0.006	0.007	0.009	0.010
	3		0.004	0.006	0.007	0.008	0.011	0.012
	4		0.005	0.007	0.008	0.010	0.012	0.013
	5		0.006	0.008	0.010	0.011	0.014	0.015
	6		0.007	0.010	0.011	0.012	0.015	0.016
	7		0.008	0.011	0.012	0.014	0.017	0.018
	8		0.009	0.012	0.014	0.015	0.018	0.019
	9		0.010	0.013	0.015	0.016	0.020	0.021
	10		0.011	0.014	0.016	0.018	0.021	0.022
	11		0.012	0.016	0.017	0.019	0.022	0.024
	12		0.013	0.017	0.019	0.020	0.024	0.025
	13		0.014	0.018	0.020	0.022	0.025	0.027
	14		0.015	0.019	0.021	0.023	0.027	0.028
	15		0.016	0.020	0.022	0.024	0.028	0.029
	16		0.018	0.021	0.024	0.025	0.029	0.031
	17		0.019	0.023	0.025	0.027	0.031	0.032
	18		0.020	0.024	0.026	0.028	0.032	0.034
	19		0.021	0.025	0.027	0.029	0.033	0.035
	20		0.022	0.026	0.028	0.030	0.035	0.036
	21		0.023	0.027	0.030	0.032	0.036	0.038
	22		0.024	0.028	0.031	0.033	0.037	0.039
	23		0.025	0.029	0.032	0.034	0.038	0.040
	24		0.026	0.031	0.033	0.035	0.040	0.041
	25		0.027	0.032	0.034	0.037	0.041	0.043
	26		0.028	0.033	0.035	0.038	0.042	0.044
	27		0.029	0.034	0.037	0.039	0.044	0.045
	28		0.030	0.035	0.038	0.040	0.045	0.047
	29		0.031	0.036	0.039	0.041	0.046	0.048
	30		0.032	0.037	0.040	0.043	0.047	0.049
	31		0.033	0.038	0.041	0.044	0.049	0.051
	32		0.034	0.040	0.042	0.045	0.050	0.052
	33		0.035	0.041	0.044	0.046	0.051	0.053
	34		0.036	0.042	0.045	0.047	0.052	0.054
	35		0.038	0.043	0.046	0.049	0.054	0.056
	36		0.039	0.044	0.047	0.050	0.055	0.057
	37		0.040	0.045	0.048	0.051	0.056	0.058
	38		0.041	0.046	0.049	0.052	0.057	0.059
	39		0.042	0.047	0.051	0.053	0.059	0.061
	40		0.043	0.049	0.052	0.054	0.060	0.062
	41		0.044	0.050	0.053	0.056	0.061	0.063
	42		0.045	0.051	0.054	0.057	0.062	0.064
	43		0.046	0.052	0.055	0.058	0.063	0.066

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.600	.900	.950	.990	.995
	44		0.047	0.053	0.056	0.059	0.065	0.067
	45		0.048	0.054	0.057	0.060	0.066	0.068
	46		0.049	0.055	0.059	0.061	0.067	0.069
	47		0.050	0.056	0.060	0.063	0.068	0.071
	48		0.051	0.057	0.061	0.064	0.070	0.072
	49		0.052	0.059	0.062	0.065	0.071	0.073
	50		0.053	0.060	0.063	0.066	0.072	0.074
	51		0.054	0.061	0.064	0.067	0.073	0.075
1000	0		0.001	0.002	0.002	0.003	0.005	0.005
	1		0.002	0.003	0.004	0.005	0.007	0.007
	2		0.003	0.004	0.005	0.006	0.008	0.009
	3		0.004	0.006	0.007	0.008	0.010	0.011
	4		0.005	0.007	0.008	0.009	0.012	0.013
	5		0.006	0.008	0.009	0.010	0.013	0.014
	6		0.007	0.009	0.011	0.012	0.015	0.016
	7		0.008	0.010	0.012	0.013	0.016	0.017
	8		0.009	0.011	0.013	0.014	0.017	0.018
	9		0.010	0.012	0.014	0.016	0.019	0.020
	10		0.011	0.014	0.015	0.017	0.020	0.021
	11		0.012	0.015	0.017	0.018	0.021	0.023
	12		0.013	0.016	0.018	0.019	0.023	0.024
	13		0.014	0.017	0.019	0.021	0.024	0.025
	14		0.015	0.018	0.020	0.022	0.025	0.027
	15		0.016	0.019	0.021	0.023	0.027	0.028
	16		0.017	0.020	0.022	0.024	0.028	0.029
	17		0.018	0.021	0.024	0.025	0.029	0.031
	18		0.019	0.022	0.025	0.027	0.030	0.032
	19		0.020	0.024	0.026	0.028	0.032	0.033
	20		0.021	0.025	0.027	0.029	0.033	0.034
	21		0.022	0.026	0.028	0.030	0.034	0.036
	22		0.023	0.027	0.029	0.031	0.035	0.037
	23		0.024	0.028	0.030	0.032	0.037	0.038
	24		0.025	0.029	0.031	0.034	0.038	0.039
	25		0.026	0.030	0.033	0.035	0.039	0.041
	26		0.027	0.031	0.034	0.036	0.040	0.042
	27		0.028	0.032	0.035	0.037	0.041	0.043
	28		0.029	0.033	0.036	0.038	0.043	0.044
	29		0.030	0.034	0.037	0.039	0.044	0.046
	30		0.031	0.035	0.038	0.040	0.045	0.047
	31		0.032	0.037	0.039	0.042	0.046	0.048

UPPER LIMITS OF BINOMIAL CONFIDENCE INTERVAL FOR DEFECTS: ONE-SIDED LIMITS

C = One-sided confidence level
N = Sample size
F = Observed number of failures in a sample of N trials

N	F	C	.500	.800	.900	.950	.990	.995
	32		0.033	0.038	0.040	0.043	0.047	0.049
	33		0.034	0.039	0.041	0.044	0.049	0.050
	34		0.035	0.040	0.043	0.045	0.050	0.052
	35		0.036	0.041	0.044	0.046	0.051	0.053
	36		0.037	0.042	0.045	0.047	0.052	0.054
	37		0.038	0.043	0.046	0.048	0.053	0.055
	38		0.039	0.044	0.047	0.050	0.055	0.056
	39		0.040	0.045	0.048	0.051	0.056	0.058
	40		0.041	0.046	0.049	0.052	0.057	0.059
	41		0.042	0.047	0.050	0.053	0.058	0.060
	42		0.043	0.048	0.051	0.054	0.059	0.061
	43		0.044	0.049	0.052	0.055	0.060	0.062
	44		0.045	0.050	0.054	0.056	0.061	0.064
	45		0.046	0.051	0.055	0.057	0.063	0.065
	46		0.047	0.052	0.056	0.058	0.064	0.066
	47		0.048	0.054	0.057	0.060	0.065	0.067
	48		0.049	0.055	0.058	0.061	0.066	0.068
	49		0.050	0.056	0.059	0.062	0.067	0.069
	50		0.051	0.057	0.060	0.063	0.068	0.071

APPENDIX 3B

Table 2

TWO-SIDED 90% CONFIDENCE LIMITS ON BINOMIAL CONFIDENCE LIMITS FOR DEFECTS

n / x	1		2		3		4		5	
0	.000	.950	.000	.776	.000	.631	.000	.527	.000	.451
1	.050	1.000	.025	.975	.017	.865	.013	.751	.010	.658
2			.224	1.000	.135	.983	.098	.902	.076	.811
3					.369	1.000	.249	.987	.189	.924
4							.473	1.000	.342	.990
5									.549	1.000

n / x	6		7		8		9		10	
0	.000	.393	.000	.348	.000	.312	.000	.263	.000	.259
1	.009	.582	.007	.521	.006	.470	.006	.429	.005	.394
2	.063	.729	.053	.659	.046	.600	.041	.550	.037	.507
3	.153	.847	.129	.775	.111	.711	.098	.655	.087	.607
4	.271	.937	.225	.871	.193	.807	.169	.749	.150	.696
5	.418	.991	.341	.947	.289	.889	.251	.831	.223	.777
6	.607	1.000	.479	.993	.400	.954	.345	.902	.304	.850
7			.652	1.000	.530	.994	.450	.959	.393	.913
8					.688	1.000	.571	.994	.493	.963
9							.717	1.000	.606	.995
10									.741	1.000

Table 2 (Continued)

x	11		12		13		14		15	
0	.000	.238	.000	.221	.000	.206	.000	.193	.000	.181
1	.005	.365	.004	.339	.004	.317	.004	.297	.003	.279
2	.033	.470	.030	.438	.028	.410	.026	.386	.024	.363
3	.079	.564	.072	.527	.066	.495	.061	.466	.057	.440
4	.135	.650	.123	.609	.113	.572	.104	.540	.097	.511
5	.200	.729	.181	.684	.166	.645	.153	.609	.142	.578
6	.271	.800	.245	.755	.224	.713	.206	.675	.191	.640
7	.350	.865	.316	.819	.287	.776	.264	.736	.244	.700
8	.436	.921	.391	.877	.355	.834	.325	.794	.300	.756
9	.530	.967	.473	.928	.428	.887	.391	.847	.360	.809
10	.635	.995	.562	.970	.505	.934	.460	.896	.422	.858
11	.762	1.000	.661	.996	.590	.972	.534	.939	.489	.903
12			.779	1.000	.683	.996	.614	.974	.560	.943
13					.794	1.000	.703	.996	.637	.976
14							.807	1.000	.721	.997
15									.819	1.000

Example: Observed from sample 5/10. The 90% confidence limits for the population are .223 and .777.

x	16		17		18		19		20	
0	.000	.171	.000	.162	.000	.153	.000	.146	.000	.139
1	.003	.264	.003	.250	.003	.238	.003	.226	.003	.216
2	.023	.344	.021	.326	.020	.310	.019	.296	.018	.282
3	.053	.417	.050	.396	.047	.377	.044	.359	.042	.344
4	.090	.484	.085	.461	.080	.439	.075	.419	.071	.401
5	.132	.549	.124	.522	.116	.498	.110	.476	.104	.455
6	.178	.608	.166	.580	.156	.554	.147	.529	.139	.507
7	.227	.667	.212	.636	.199	.608	.188	.582	.177	.558
8	.279	.721	.260	.689	.244	.659	.229	.632	.217	.606
9	.333	.773	.311	.740	.291	.709	.274	.679	.259	.653
10	.392	.822	.364	.788	.341	.756	.321	.726	.302	.698
11	.451	.868	.420	.834	.392	.801	.368	.771	.347	.741
12	.516	.910	.478	.876	.446	.844	.418	.812	.394	.783
13	.583	.947	.539	.915	.502	.884	.471	.853	.442	.823
14	.656	.977	.604	.950	.561	.920	.524	.890	.493	.861
15	.736	.997	.674	.979	.625	.953	.581	.925	.545	.896
16	.829	1.000	.750	.997	.690	.980	.641	.956	.599	.929
17			.838	1.000	.762	.997	.704	.981	.656	.958
18					.847	1.000	.774	.997	.718	.982
19							.854	1.000	.784	.997
20									.861	1.000

Example: Observed from sample 10/20. The 90% confidence limits for the population are .302 and .698.

Table 2 (Continued)

x \ n	22		24		26		28		30	
0	.000	.127	.000	.117	.000	.109	.000	.101	.000	.095
1	.002	.198	.002	.183	.002	.170	.002	.158	.002	.149
2	.016	.260	.015	.240	.014	.223	.013	.208	.012	.196
3	.038	.316	.035	.292	.032	.272	.030	.254	.028	.238
4	.065	.370	.059	.342	.054	.318	.050	.297	.047	.280
5	.094	.420	.086	.389	.079	.362	.073	.339	.068	.319
6	.124	.468	.115	.435	.106	.406	.098	.379	.091	.357
7	.160	.515	.146	.479	.134	.447	.124	.419	.115	.394
8	.196	.561	.178	.522	.163	.487	.151	.457	.140	.429
9	.233	.605	.211	.563	.194	.526	.179	.493	.166	.466
10	.271	.647	.246	.603	.226	.564	.208	.531	.193	.499
11	.311	.689	.282	.643	.259	.602	.238	.565	.221	.533
12	.353	.729	.319	.681	.292	.638	.270	.600	.249	.567
13	.395	.767	.357	.718	.327	.673	.301	.633	.279	.597
14	.439	.804	.397	.754	.362	.708	.333	.667	.308	.630
15	.485	.840	.437	.789	.398	.741	.367	.699	.339	.661
16	.532	.876	.478	.822	.436	.774	.400	.730	.370	.692
17	.580	.906	.521	.854	.474	.806	.435	.762	.403	.721
18	.630	.935	.565	.885	.513	.837	.469	.792	.433	.751
19	.684	.962	.611	.914	.553	.866	.507	.821	.467	.779
20	.740	.984	.658	.941	.594	.894	.543	.849	.501	.807
21	.802	.998	.708	.965	.638	.921	.581	.876	.534	.834
22	.873	1.000	.760	.985	.682	.946	.621	.902	.571	.860
23			.817	.998	.728	.968	.661	.927	.606	.885
24			.883	1.000	.777	.986	.703	.950	.643	.909
25					.830	.998	.746	.970	.681	.932
26					.891	1.000	.792	.987	.720	.953
27							.842	.998	.762	.972
28							.899	1.000	.804	.988
29									.851	.998
30									.905	1.000

Example: Observed from sample 6/30. The 90% confidence limits for the population are .091 and .357.

APPENDIX 3B

Table 2

TWO-SIDED 90% CONFIDENCE LIMITS ON BINOMIAL p

x \ n	35		40	
0	.000	.082	.000	.072
1	.001	.128	.001	.113
2	.010	.169	.009	.149
3	.024	.206	.021	.183
4	.040	.243	.035	.215
5	.058	.277	.051	.245
6	.078	.311	.067	.275
7	.098	.343	.085	.304
8	.119	.374	.103	.331
9	.141	.405	.123	.360
10	.163	.436	.142	.387
11	.187	.467	.162	.414
12	.211	.496	.184	.441
13	.235	.524	.204	.466
14	.261	.553	.226	.492
15	.286	.581	.247	.518
16	.311	.609	.269	.543
17	.337	.636	.292	.567
18	.364	.663	.314	.592
19	.391	.689	.333	.615
20	.419	.714	.362	.638
21	.447	.739	.385	.662
22	.476	.765	.408	.686
23	.504	.789	.433	.708
24	.533	.813	.457	.731
25	.564	.837	.482	.753

x \ n	35		40	
26	.595	.859	.508	.774
27	.626	.881	.534	.796
28	.657	.902	.559	.816
29	.689	.922	.586	.838
30	.723	.942	.613	.858
31	.757	.960	.640	.877
32	.794	.976	.669	.897
33	.831	.990	.696	.915
34	.872	.999	.725	.933
35	.918	1.000	.755	.949
36			.785	.965
37			.817	.979
38			.851	.991
39			.887	.999
40			.928	1.000
41				
42				
43				
44				
45				
46				
47				
48				
49				
50				

Example: Observed from sample 35/50. The 90% confidence limits for the population are .576 and .805.

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APPENDIX 3B

Table 2 (Continued)

x \ n	45		50		x \ n	45		50	
0	.000	.064	.000	.058					
1	.001	.101	.001	.091	26	.445	.704	.395	.643
2	.008	.133	.007	.120	27	.467	.723	.414	.662
3	.018	.163	.017	.148	28	.488	.742	.435	.680
4	.031	.192	.028	.174	29	.511	.762	.455	.698
5	.045	.220	.040	.199	30	.533	.782	.475	.718
6	.060	.246	.054	.223	31	.558	.801	.494	.734
7	.075	.273	.067	.247	32	.581	.820	.514	.753
8	.092	.297	.082	.270	33	.603	.838	.536	.771
9	.108	.323	.097	.293	34	.628	.856	.556	.788
10	.126	.348	.113	.316	35	.652	.874	.576	.805
11	.144	.372	.129	.337	36	.677	.892	.597	.822
12	.162	.397	.145	.359	37	.703	.908	.619	.839
13	.180	.419	.161	.381	38	.727	.925	.641	.855
14	.199	.442	.178	.403	39	.754	.940	.663	.871
15	.218	.467	.195	.424	40	.780	.955	.684	.887
16	.238	.489	.212	.444	41	.808	.969	.707	.903
17	.258	.512	.229	.464	42	.837	.982	.730	.918
18	.277	.533	.247	.486	43	.867	.992	.753	.933
19	.296	.555	.266	.506	44	.899	.999	.777	.946
20	.317	.576	.282	.525	45	.936	1.000	.801	.960
21	.337	.598	.302	.545	46			.826	.972
22	.359	.621	.320	.565	47			.852	.983
23	.379	.641	.338	.586	48			.880	.993
24	.402	.663	.357	.605	49			.909	.999
25	.424	.683	.375	.625	50			.942	1.000

Example: Observed from sample 35/50. The 90% confidence limits for the population are .576 and .805.

Table 2 (Continued)

TWO-SIDED 90% CONFIDENCE LIMITS ON BINOMIAL p

n = 60

x			x			x			x		
0	.000	.049									
1	.001	.076	16	.175	.376	31	.403	.628	46	.659	.853
2	.006	.101	17	.189	.394	32	.419	.643	47	.677	.867
3	.014	.124	18	.204	.412	33	.437	.660	48	.696	.880
4	.023	.146	19	.218	.429	34	.453	.677	49	.715	.894
5	.033	.167	20	.233	.445	35	.470	.691	50	.734	.907
6	.045	.187	21	.248	.463	36	.485	.708	51	.752	.920
7	.056	.208	22	.263	.481	37	.502	.722	52	.772	.932
8	.068	.228	23	.278	.498	38	.519	.737	53	.792	.944
9	.080	.248	24	.292	.515	39	.537	.752	54	.813	.955
10	.093	.266	25	.309	.530	40	.555	.767	55	.833	.967
11	.106	.285	26	.323	.547	41	.571	.782	56	.854	.977
12	.120	.304	27	.340	.563	42	.588	.796	57	.876	.986
13	.133	.323	28	.357	.581	43	.606	.811	58	.899	.994
14	.147	.341	29	.374	.597	44	.624	.825	59	.924	.999
15	.161	.358	30	.387	.613	45	.642	.839	60	.951	1.000

APPENDIX 3B

Table 2 (Continued)

n = 80

x			x			x			x		
0	.000	.037									
1	.001	.058	21	.183	.356	41	.414	.610	61	.672	.838
2	.004	.077	22	.194	.368	42	.428	.621	62	.685	.849
3	.010	.094	23	.205	.382	43	.440	.633	63	.698	.860
4	.017	.111	24	.216	.395	44	.452	.644	64	.712	.870
5	.025	.127	25	.228	.408	45	.465	.657	65	.726	.881
6	.033	.143	26	.240	.422	46	.477	.669	66	.741	.891
7	.042	.158	27	.250	.436	47	.490	.680	67	.754	.901
8	.051	.173	28	.262	.447	48	.503	.692	68	.768	.911
9	.060	.188	29	.274	.460	49	.515	.703	69	.783	.921
10	.069	.203	30	.284	.472	50	.528	.716	70	.797	.931
11	.079	.217	31	.297	.485	51	.540	.726	71	.812	.940
12	.089	.232	32	.308	.497	52	.553	.738	72	.827	.949
13	.099	.246	33	.320	.510	53	.564	.750	73	.842	.958
14	.109	.259	34	.331	.523	54	.578	.760	74	.857	.967
15	.119	.274	35	.343	.535	55	.592	.772	75	.873	.975
16	.130	.288	36	.356	.548	56	.605	.784	76	.889	.983
17	.140	.302	37	.367	.560	57	.618	.795	77	.906	.990
18	.151	.315	38	.379	.572	58	.632	.806	78	.923	.996
19	.162	.328	39	.390	.586	59	.644	.817	79	.942	.999
20	.172	.343	40	.402	.598	60	.657	.828	80	.963	1.000

Example: Observed from sample 50/80. The 90% confidence limits for the population are .528 and .716.

n = 100

x			x			x			x		
0	.000	.000									
1	.001	.047	26	.190	.343	51	.423	.596	76	.679	.828
2	.004	.061	27	.198	.353	52	.433	.605	77	.690	.838
3	.008	.075	28	.207	.364	53	.443	.615	78	.702	.846
4	.014	.089	29	.215	.373	54	.453	.626	79	.712	.854
5	.020	.102	30	.224	.384	55	.462	.635	80	.723	.863
6	.027	.115	31	.234	.395	56	.472	.645	81	.734	.872
7	.033	.127	32	.244	.405	57	.482	.654	82	.745	.881
8	.040	.140	33	.252	.415	58	.492	.665	83	.756	.889
9	.048	.152	34	.262	.426	59	.503	.674	84	.767	.897
10	.055	.164	35	.271	.437	60	.513	.683	85	.779	.905
11	.063	.176	36	.281	.446	61	.523	.692	86	.790	.914
12	.071	.187	37	.289	.456	62	.534	.702	87	.801	.921
13	.079	.199	38	.298	.466	63	.544	.711	88	.813	.929
14	.086	.210	39	.308	.477	64	.554	.719	89	.824	.937
15	.095	.221	40	.317	.487	65	.563	.729	90	.836	.945
16	.103	.233	41	.326	.497	66	.574	.738	91	.848	.952
17	.111	.244	42	.335	.508	67	.585	.748	92	.860	.960
18	.119	.255	43	.346	.518	68	.595	.756	93	.873	.967
19	.128	.266	44	.355	.528	69	.605	.766	94	.885	.973
20	.137	.277	45	.365	.538	70	.616	.776	95	.898	.980
21	.146	.288	46	.374	.547	71	.627	.785	96	.911	.986
22	.154	.298	47	.385	.557	72	.636	.793	97	.925	.992
23	.162	.310	48	.395	.567	73	.647	.802	98	.939	.996
24	.172	.321	49	.404	.577	74	.657	.810	99	.953	.999
25	.181	.331	50	.414	.586	75	.669	.819	100	.970	1.000

Example: Observed from sample 50/100. The 90% confidence limits for the population are .414 and .586.

Table 2 (Continued)

TWO-SIDED 95% CONFIDENCE INTERVALS FOR BINOMIAL DISTRIBUTION

The following lists the 95% confidence interval for the binomial distribution. These tables are similar to the 90% tables.

TWO-SIDED 95% CONFIDENCE LIMITS FOR DEFECTS

x \ n	1		2		3		4		5	
0	.000	.975	.000	.842	.000	.708	.000	.602	.000	.522
1	.025	1.000	.013	.937	.008	.906	.006	.806	.005	.716
2			.158	1.000	.094	.992	.068	.932	.053	.853
3					.292	1.000	.194	.994	.147	.947
4							.398	1.000	.234	.995
5									.478	1.000

x \ n	6		7		8		9		10	
0	.000	.459	.000	.410	.000	.369	.000	.336	.000	.308
1	.004	.641	.004	.579	.003	.527	.003	.483	.003	.445
2	.043	.777	.037	.710	.032	.651	.028	.600	.025	.556
3	.118	.882	.099	.816	.085	.755	.075	.701	.067	.652
4	.223	.957	.184	.901	.157	.843	.137	.788	.122	.738
5	.359	.996	.290	.963	.245	.915	.212	.863	.187	.813
6		1.000	.421	.996	.349	.968	.299	.925	.262	.878
7			.590	1.000	.473	.997	.400	.972	.348	.933
8					.631	1.000	.517	.997	.444	.975
9							.664	1.000	.555	.997
10									.692	1.000

Table 2 (Continued)

x \ n	11		12		13		14		15	
0	.000	.285	.000	.265	.000	.247	.000	.232	.000	.218
1	.002	.413	.002	.385	.002	.360	.002	.339	.002	.319
2	.023	.518	.021	.484	.019	.454	.018	.428	.017	.405
3	.060	.610	.055	.572	.050	.538	.047	.508	.043	.481
4	.109	.692	.099	.651	.091	.614	.084	.581	.078	.551
5	.167	.766	.151	.723	.139	.684	.128	.649	.118	.616
6	.234	.833	.211	.789	.192	.749	.177	.711	.163	.677
7	.308	.891	.277	.849	.251	.808	.230	.770	.213	.734
8	.390	.940	.349	.901	.316	.861	.289	.823	.266	.787
9	.482	.977	.428	.945	.386	.909	.351	.872	.323	.837
10	.587	.998	.516	.979	.462	.950	.419	.916	.384	.882
11	.715	1.000	.615	.998	.546	.981	.492	.953	.449	.922
12			.735	1.000	.640	.998	.572	.982	.519	.957
13					.753	1.000	.661	.998	.595	.983
14							.768	1.000	.681	.998
15									.782	1.000

Example: Observed from sample 5/10. The 95% confidence limits for the population are .187 and .813.

Table 2 (Continued)

x \ n	16		17		18		19		20	
0	.000	.206	.000	.195	.000	.185	.000	.176	.000	.168
1	.002	.302	.001	.287	.001	.273	.001	.260	.001	.249
2	.016	.383	.015	.364	.014	.347	.013	.331	.012	.317
3	.040	.456	.038	.434	.036	.414	.034	.396	.032	.379
4	.073	.524	.068	.499	.064	.476	.061	.456	.057	.437
5	.110	.587	.103	.560	.097	.535	.091	.512	.087	.491
6	.152	.646	.142	.617	.133	.590	.126	.565	.119	.543
7	.198	.701	.184	.671	.173	.643	.163	.616	.154	.592
8	.247	.753	.230	.722	.215	.692	.203	.665	.191	.639
9	.299	.802	.278	.770	.260	.740	.244	.711	.231	.685
10	.354	.848	.329	.816	.308	.785	.289	.756	.272	.728
11	.413	.890	.383	.858	.357	.827	.335	.797	.315	.769
12	.476	.927	.440	.897	.410	.867	.384	.837	.361	.809
13	.544	.960	.501	.932	.465	.903	.435	.874	.408	.846
14	.617	.984	.566	.962	.524	.936	.388	.909	.457	.881
15	.698	.998	.636	.985	.586	.964	.544	.939	.509	.913
16	.794	1.000	.713	.999	.653	.986	.604	.966	.563	.943
17			.805	1.000	.727	.999	.669	.987	.621	.968
18					.815	1.000	.740	.999	.683	.988
19							.824	1.000	.751	.999
20									.832	1.000

x \ n	21		22		23		24		25	
0	.000	.161	.000	.154	.000	.148	.000	.142	.000	.137
1	.001	.238	.001	.229	.001	.219	.001	.211	.001	.203
2	.012	.304	.011	.292	.011	.281	.010	.270		
3	.030	.363	.029	.349	.028	.336	.027	.323	.025	.312
4	.054	.419	.052	.403	.050	.388	.047	.374		
5	.082	.471	.078	.453	.075	.436	.071	.421	.068	.407
6	.113	.522	.107	.502	.102	.484	.098	.467	.094	.451
7	.146	.570	.139	.549	.132	.529	.126	.512	.121	.494
8	.181	.616	.172	.593	.164	.573	.156	.553	.149	.535
9	.218	.660	.207	.636	.197	.615	.188	.594	.180	.575
10	.257	.702	.244	.678	.232	.655	.221	.634	.211	.614
11	.298	.743	.282	.718	.268	.694	.256	.672	.244	.651
12	.340	.782	.322	.756	.306	.732	.291	.709	.278	.687
13	.384	.819	.364	.793	.345	.768	.328	.744	.313	.722
14	.430	.854	.407	.828	.385	.803	.366	.779	.349	.756
15	.478	.887	.451	.861	.427	.836	.406	.812	.386	.789
16	.529	.918	.498	.893	.471	.868	.447	.844	.425	.820
17	.581	.946	.547	.922	.516	.898	.488	.874	.465	.851
18	.637	.970	.597	.948	.564	.925	.533	.902	.506	.879
19	.696	.988	.651	.971	.612	.950	.579	.929	.549	.906
20	.762	.999	.708	.989	.664	.972	.626	.953	.593	.932
21	.839	1.000	.771	.999	.719	.989	.677	.973		
22			.846	1.000	.781	.999	.730	.990	.688	.975
23					.852	1.000	.789	.999		
24							.858	1.000	.797	.999
25									.863	1.000

Example: Observed from sample 10/25. The 95% confidence limits for the population are .211 and .614.

APPENDIX 3B

Table 2 (Continued)

x \ n	26		28		30		35		40	
0	.000	.132	.000	.123	.000	.116	.000	.100	.000	.088
1	.001	.197	.001	.184	.001	.172	.001	.149	.001	.132
2	.009	.251	.009	.235	.008	.221	.007	.192	.006	.169
3	.024	.301	.023	.282	.021	.265	.018	.230	.016	.204
4	.044	.349	.040	.327	.038	.307	.032	.268	.028	.236
5	.066	.393	.061	.369	.056	.343	.048	.303	.042	.268
6	.090	.436	.083	.410	.077	.386	.066	.336	.057	.298
7	.116	.478	.107	.449	.099	.423	.084	.369	.073	.328
8	.143	.518	.132	.487	.123	.459	.104	.401	.090	.357
9	.172	.557	.159	.524	.148	.494	.125	.433	.109	.385
10	.202	.595	.186	.560	.173	.528	.147	.463	.127	.412
11	.234	.631	.215	.594	.199	.561	.169	.493	.146	.439
12	.266	.666	.245	.628	.227	.594	.192	.522	.166	.465
13	.299	.701	.275	.661	.255	.626	.215	.551	.185	.491
14	.334	.734	.306	.694	.283	.657	.239	.578	.206	.517
15	.369	.766	.339	.725	.313	.687	.263	.607	.227	.542
16	.405	.798	.372	.755	.343	.717	.288	.634	.249	.567
17	.443	.828	.406	.785	.374	.745	.314	.660	.271	.590
18	.482	.857	.440	.814	.406	.773	.340	.686	.293	.615
19	.522	.884	.476	.841	.439	.801	.366	.712	.315	.639
20	.564	.910	.513	.868	.472	.827	.393	.737	.338	.662
21	.607	.934	.551	.893	.506	.852	.422	.761	.361	.685
22	.651	.956	.590	.917	.541	.877	.449	.785	.385	.707
23	.699	.976	.631	.939	.577	.901	.478	.808	.410	.729
24	.749	.991	.673	.960	.614	.923	.507	.831	.433	.751
25	.803	.999	.718	.977	.652	.944	.537	.853	.458	.773
26	.863	1.000	.765	.991	.693	.962	.567	.875	.483	.794
27			.816	.999	.735	.979	.599	.896	.509	.815
28			.877	1.000	.779	.992	.631	.916	.535	.834
29					.828	.999	.664	.934	.561	.854
30					.884	1.000	.697	.952	.588	.873
31							.732	.968	.615	.891
32							.770	.982	.643	.910
33							.808	.993	.672	.927
34							.851	.999	.702	.943
35							.900	1.000	.732	.958
36									.764	.972
37									.796	.984
38									.831	.994
39									.868	.999
40									.912	1.000

Example: Observed from sample 25/40. The 95% confidence limits for the population are .458 and .773.

n = 50

x		
0	.000	.071
1	.C01	.1C6
2	.005	.137
3	.C13	.165
4	.C22	.192
5	.C33	.218
6	.C45	.243
7	.C58	.267
8	.C72	.291
9	.086	.314
10	.1C0	.338
11	.115	.360
12	.131	.381
13	.146	.404
14	.163	.424
15	.179	.446
16	.195	.467
17	.212	.488
18	.229	.5C8
19	.247	.527
20	.264	.548
21	.282	.568
22	.3C0	.588
23	.318	.6C6
24	.337	.626
25	.356	.644

x		
26	.374	.663
27	.394	.682
28	.412	.700
29	.432	.718
30	.452	.736
31	.473	.753
32	.492	.771
33	.512	.788
34	.533	.805
35	.554	.821
36	.576	.837
37	.596	.854
38	.619	.869
39	.64C	.885
40	.662	.900
41	.686	.914
42	.7C9	.928
43	.733	.942
44	.757	.955
45	.782	.967
46	.8C8	.978
47	.835	.987
48	.863	.995
49	.894	.999
50	.929	1.000

Example: Observed from sample 15/50. The 95% confidence limits for the population are .179 and .446.

n = 100

x			x						x		
0	.000	.036									
1	.000	.054	26	.177	.357	51	.408	.611	76	.664	.839
2	.002	.070	27	.187	.368	52	.418	.620	77	.676	.848
3	.006	.085	28	.195	.378	53	.427	.630	78	.686	.856
4	.011	.099	29	.204	.390	54	.437	.639	79	.697	.865
5	.016	.113	~~	.213	.399	55	.447	.650	80	.708	.874
6	.022	.126	31	.221	.410	56	.457	.659	81	.719	.881
7	.029	.139		.230	.420	57	.467	.668	82	.731	.890
8	.035	.152	∴	.240	.431	58	.477	.678	83	.742	.898
9	.042	.164	34	.248	.441	59	.487	.687	84	.753	.906
10	.049	.176	35	.257	.452	60	.497	.697	85	.764	.914
11	.056	.188	36	.266	.463	61	.507	.706	86	.777	.922
12	.064	.200	37	.276	.472	62	.518	.716	87	.788	.929
13	.071	.212	38	.284	.482	63	.528	.724	88	.800	.936
14	.078	.223	39	.294	.493	64	.537	.734	89	.812	.944
15	.086	.236	40	.303	.503	65	.548	.743	90	.824	.951
16	.094	.247	41	.313	.513	66	.559	.752	91	.836	.958
17	.102	.258	42	.322	.523	67	.569	.760	92	.848	.965
18	.110	.269	43	.332	.533	68	.580	.770	93	.861	.971
19	.119	.281	44	.341	.543	69	.590	.779	94	.874	.978
20	.126	.292	45	.350	.553	70	.601	.787	95	.887	.984
21	.135	.303	46	.361	.563	71	.610	.796	96	.901	.989
22	.144	.314	47	.370	.573	72	.622	.805	97	.915	.994
23	.152	.324	48	.380	.582	73	.632	.813	98	.930	.998
24	.161	.336	49	.389	.592	74	.643	.823	99	.946	1.000
25	.169	.347	50	.398	.602	75	.653	.831	100	.964	1.000

Example: Observed from sample 50/100. The 95% confidence limits for the population are .398 and .602.

Table 2 (Continued)

99% CONFIDENCE INTERFVAL FOR BINOMIAL DISTRIBUTION

The following lists the 99% confidence interval for the binomial distribution. These tables are similar to the 90% tables.

TWO-SIDED 99% CONFIDENCE LIMITS FOR DEFECTS

x \ n	1		2		3		4		5	
0	.000	.995	.000	.929	.000	.829	.000	.734	.000	.653
1	.005	1.000	.003	.997	.002	.959	.001	.889	.001	.815
2			.071	1.000	.041	.998	.029	.971	.023	.917
3					.171	1.000	.111	.999	.083	.977
4							.266	1.000	.185	.999
5									.347	1.000

x \ n	6		7		8		9		10	
0	.000	.586	.000	.531	.000	.484	.000	.445	.000	.411
1	.001	.746	.001	.685	.001	.632	.001	.585	.001	.544
2	.019	.856	.016	.797	.014	.742	.012	.693	.011	.648
3	.066	.934	.055	.882	.047	.830	.042	.781	.037	.735
4	.144	.981	.118	.945	.100	.900	.087	.854	.077	.809
5	.254	.999	.203	.984	.170	.953	.146	.913	.128	.872
6	.414	1.000	.315	.999	.258	.986	.219	.958	.191	.923
7			.469	1.000	.368	.999	.307	.988	.265	.963
8					.516	1.000	.415	.999	.352	.989
9							.555	1.000	.456	.999
10									.589	1.000

Table 2 (Continued)

x \ n	11		12		13		14		15	
0	.000	.382	.000	.357	.000	.335	.000	.315	.000	.298
1	.000	.509	.000	.477	.000	.449	.000	.424	.000	.402
2	.010	.608	.009	.573	.008	.541	.008	.512	.007	.486
3	.033	.693	.030	.655	.028	.621	.026	.589	.024	.561
4	.069	.767	.062	.728	.057	.691	.053	.658	.049	.627
5	.114	.831	.103	.791	.094	.755	.087	.720	.080	.688
6	.169	.886	.152	.848	.138	.811	.127	.777	.117	.744
7	.233	.931	.209	.897	.189	.862	.172	.828	.159	.795
8	.307	.967	.272	.938	.245	.906	.223	.873	.205	.841
9	.392	.990	.345	.970	.309	.943	.280	.913	.256	.883
10	.491	1.000	.427	.991	.379	.972	.342	.947	.312	.920
11	.618	1.000	.523	1.000	.459	.992	.411	.974	.373	.951
12			.643	1.000	.551	1.000	.488	.992	.439	.976
13					.665	1.000	.576	1.000	.514	.993
14							.685	1.000	.598	1.000
15									.702	1.000

Example: Observed from sample 5/10. The 99% confidence limits for the population are .128 and .872.

x \ n	16		17		18		19		20	
0	.000	.282	.000	.268	.000	.255	.000	.243	.000	.233
1	.000	.381	.000	.363	.000	.346	.000	.331	.000	.317
2	.007	.463	.006	.441	.006	.422	.006	.404	.005	.387
3	.022	.534	.021	.510	.020	.488	.019	.468	.018	.449
4	.045	.599	.043	.573	.040	.549	.038	.527	.036	.507
5	.075	.658	.070	.631	.065	.605	.062	.582	.058	.560
6	.109	.714	.101	.685	.095	.658	.090	.633	.085	.610
7	.147	.764	.137	.735	.128	.707	.121	.681	.114	.657
8	.189	.811	.176	.781	.165	.753	.155	.726	.146	.701
9	.236	.853	.219	.824	.205	.795	.192	.768	.181	.743
10	.286	.891	.265	.863	.247	.835	.232	.808	.218	.782
11	.342	.925	.315	.899	.293	.872	.274	.845	.257	.819
12	.401	.955	.369	.930	.342	.905	.319	.879	.299	.854
13	.466	.978	.427	.957	.395	.935	.367	.910	.343	.886
14	.537	.993	.490	.979	.451	.960	.418	.938	.390	.915
15	.619	1.000	.559	.994	.512	.980	.473	.962	.440	.942
16	.718	1.000	.637	1.000	.578	.994	.532	.981	.493	.964
17			.732	1.000	.654	1.000	.596	.994	.551	.982
18					.745	1.000	.669	1.000	.613	.995
19							.757	1.000	.683	1.000
20									.767	1.000

Example: Observed from sample 10/20. The 99% confidence limits for the population are .218 and .782.

APPENDIX 3B

Table 2 (Continued)

x \ n	22		24		26		28		30	
0	.000	.214	.000	.198	.000	.184	.000	.172	.000	.162
1	.000	.292	.000	.271	.000	.253	.000	.237	.000	.223
2	.005	.358	.004	.332	.004	.310	.004	.291	.004	.274
3	.016	.416	.015	.387	.013	.362	.012	.340	.012	.320
4	.032	.470	.029	.438	.027	.410	.025	.385	.023	.363
5	.053	.520	.048	.485	.044	.455	.041	.428	.038	.404
6	.076	.567	.069	.531	.064	.498	.059	.469	.054	.443
7	.102	.612	.093	.573	.085	.538	.078	.508	.073	.480
8	.131	.655	.119	.614	.109	.578	.100	.545	.093	.516
9	.162	.695	.146	.653	.134	.615	.123	.581	.114	.550
10	.195	.734	.176	.690	.161	.651	.148	.616	.137	.583
11	.229	.771	.207	.726	.189	.686	.173	.649	.160	.616
12	.266	.805	.240	.760	.218	.719	.200	.682	.185	.647
13	.305	.838	.274	.793	.249	.751	.228	.713	.211	.678
14	.345	.869	.310	.824	.281	.782	.257	.743	.237	.707
15	.388	.898	.347	.854	.314	.811	.287	.772	.265	.735
16	.433	.924	.386	.881	.349	.839	.318	.800	.293	.763
17	.480	.947	.427	.907	.385	.866	.351	.827	.322	.789
18	.530	.968	.469	.931	.422	.891	.384	.852	.353	.815
19	.584	.984	.515	.952	.462	.915	.419	.877	.384	.840
20	.642	.995	.562	.971	.502	.936	.455	.900	.417	.863
21	.708	1.000	.613	.985	.545	.956	.492	.922	.450	.886
22	.786	1.000	.668	.996	.590	.973	.531	.941	.484	.907
23			.729	1.000	.638	.987	.572	.959	.520	.927
24			.802	1.000	.690	.996	.615	.975	.557	.946
25					.747	1.000	.660	.988	.596	.962
26					.816	1.000	.709	.996	.637	.977
27							.763	1.000	.680	.988
28							8.28	1.000	.726	.996
29									.777	1.000
30									.838	1.000

Example: Observed from sample 6/30. The 99% confidence limits for the population are .054 and .443.

Table 2 (Continued)

x \ n	35		40			x \ n	35		40	
0	.000	.140	.000	.124		21	.371	.802	.317	.727
						22	.398	.823	.339	.748
1	.000	.194	.000	.172		23	.426	.844	.362	.769
2	.003	.239	.003	.212		24	.455	.865	.386	.789
3	.010	.280	.009	.249		25	.484	.885	.411	.809
4	.020	.318	.017	.283						
5	.032	.354	.028	.315		26	.515	.903	.434	.829
						27	.545	.921	.459	.847
6	.046	.389	.040	.346		28	.578	.938	.485	.866
7	.062	.422	.054	.376		29	.611	.954	.511	.883
8	.079	.455	.068	.406		30	.646	.968	.539	.900
9	.097	.485	.084	.434						
10	.115	.516	.100	.461		31	.682	.980	.566	.916
						32	.720	.990	.594	.932
11	.135	.545	.117	.489		33	.761	.997	.624	.946
12	.156	.574	.134	.515		34	.806	1.000	.654	.960
13	.177	.602	.153	.541		35	.860	1.000	.685	.972
14	.198	.629	.177	.566						
15	.222	.655	.191	.589		36			.717	.983
						37			.751	.991
16	.245	.681	.211	.614		38			.788	.997
17	.269	.706	.231	.638		39			.828	1.000
18	.294	.731	.252	.661		40			.876	1.000
19	.319	.755	.273	.683						
20	.345	.778	.295	.705						

Table 2 (Continued)

x \ n	45		50	
0	.000	.111	.000	.101
1	.000	.154	.000	.139
2	.002	.190	.002	.173
3	.008	.223	.007	.203
4	.015	.254	.014	.231
5	.025	.284	.022	.258
6	.036	.312	.032	.285
7	.047	.339	.042	.309
8	.060	.366	.054	.334
9	.074	.392	.066	.357
10	.088	.418	.079	.381
11	.103	.442	.092	.404
12	.118	.465	.106	.425
13	.134	.490	.120	.447
14	.150	.513	.134	.469
15	.167	.536	.149	.490
16	.184	.558	.164	.511
17	.202	.580	.180	.532
18	.221	.601	.196	.551
19	.239	.623	.213	.572
20	.257	.644	.229	.591

x \ n	45		50	
21	.276	.664	.246	.610
22	.296	.684	.263	.629
23	.316	.704	.280	.648
24	.336	.724	.298	.666
25	.356	.743	.315	.685
26	.377	.761	.334	.702
27	.399	.779	.352	.720
28	.420	.798	.371	.737
29	.442	.816	.390	.754
30	.464	.833	.409	.771
31	.487	.850	.428	.787
32	.510	.866	.449	.804
33	.535	.882	.468	.820
34	.558	.897	.489	.836
35	.582	.912	.510	.851
36	.608	.926	.531	.866
37	.634	.940	.553	.880
38	.661	.953	.575	.894
39	.688	.964	.596	.908
40	.716	.975	.619	.921
41	.746	.985	.643	.934
42	.777	.992	.666	.946
43	.810	.998	.691	.958
44	.846	1.000	.715	.968
45	.889	1.000	.742	.978
46			.769	.986
47			.797	.993
48			.827	.998
49			.861	1.000
50			.899	1.000

Example: Observed from sample 35/50. The 99% confidence limits
for the population are .510 and .851.

Table 2 (Continued)

$n = 60$

x		
C	.CCC	.085
1	.CCG	.117
2	.CC2	.146
3	.CC6	.172
4	.C11	.195
5	.C18	.218
6	.C26	.241
7	.C35	.263
8	.C45	.283
9	.C55	.3C4
1C	.C65	.324
11	.C76	.343
12	.C8'7	.363
13	.098	.381
14	.110	399
15	.123	.418
16	.135	.437
17	.148	.454
18	.160	.472
19	.174	.489
2C	.187	.5C7
21	.2C1	.524
22	.215	.5.0
23	.228	.557
24	.243	.574
25	.257	.59C
26	.272	.606
27	.286	.622
28	.3C1	.637
29	.316	.654
30	.331	.669

x		
31	.346	.684
32	.363	.699
33	.378	.714
34	.394	.728
35	.410	.743
36	.426	.757
37	.443	.772
38	.460	.785
39	.476	.799
40	.493	.813
41	.511	.826
42	.528	.840
43	.546	.852
44	.563	.865
45	.582	.877
46	.601	.890
47	.619	.9C2
48	.637	.913
49	.657	.924
5C	.676	.935
51	.696	.945
52	.717	.955
53	.737	.965
54	.759	.974
55	.782	.982
56	.805	.989
57	.828	.994
58	.854	.998
59	.883	1.000
60	.915	1.CC0

APPENDIX 3B

Table 2 (Continued)

n = 80

x			x			x		
0	.CCO	.C64	26	.198	.474	56	.553	.822
			27	.2C8	.486	57	.5C6	.833
1	.CCC	.C89	28	.219	.5CO	58	.579	.844
2	.CC1	.111	29	.230	.513	59	.593	.853
3	.CC4	.131	30	.241	.525	60	.6C6	.863
4	.CC9	.149						
5	.C14	.167	31	.251	.538	61	.621	.872
			32	.262	.55C	62	.634	.882
6	.C2C	.184	33	.273	.561	63	.648	.891
7	.C26	.2C1	34	.284	.574	64	.662	.9C1
8	.C33	.217	35	.296	.587	65	.677	.91C
9	.C4C	.233						
10	.C48	.249	36	.3C7	.598	66	.691	.918
			37	.318	.611	67	.7C5	.927
11	.C56	.264	38	.331	.623	68	.720	.936
12	.C64	.280	39	.342	.635	69	.736	.944
13	.C73	.295	40	.354	.646	70	.751	.952
14	.C82	.3C9						
15	.C9C	.323	41	.365	.658	71	.767	.960
			42	.377	.669	72	.783	.967
16	.C99	.338	43	.389	.682	73	.799	.974
17	.1C9	.352	44	.402	.693	74	.816	.98C
18	.118	.366	45	.413	.704	75	.833	.986
19	.128	.379						
20	.137	.394	46	.426	.716	76	.851	.991
			47	.439	.727	77	.869	.996
21	.147	.407	48	.450	.738	78	.889	.999
22	.156	.421	49	.462	.749	79	.911	1.0CC
23	.167	.434	50	.475	.759	80	.936	1.0CO
24	.178	.447						
25	.188	.461	51	.487	.770			
			52	.5C0	.781			
			53	.514	.792			
			54	.526	.8C2			
			55	.539	.812			

Example: Observed from sample 50/80. The 99% confidence limits for the population are .475 and .759.

Table 2 (Continued)

$$n = 100$$

x			x			x		
C	.000	.052	36	.240	.493	66	.527	.777
			37	.249	.503	67	.538	.786
1	000	.072	38	.259	.514	68	.543	.794
2	001	.089	39	.268	.523	69	.559	.803
3	.003	.105	40	.276	.534	70	.569	.811
4	.007	.120						
5	.011	.135	41	.286	.543	71	.581	.820
			42	.294	.553	72	.591	.828
6	.016	.149	43	.304	.563	73	.601	.836
7	.021	.163	44	.312	.573	74	.612	.844
8	.026	.176	45	.322	.583	75	.622	.852
9	.032	.189						
10	.038	.202	46	.331	.592	76	.633	.860
			47	.341	.602	77	.644	.868
11	.044	.214	48	.350	.611	78	.656	.876
12	.051	.227	49	.359	.622	79	.667	.884
13	.058	.240	50	.369	.631	80	.679	.891
14	.065	.251						
15	.072	.263	51	.378	.641	81	.690	.899
			52	.389	.650	82	.702	.907
16	.079	.275	53	.398	.659	83	.714	.914
17	.086	.286	54	.408	.669	84	.725	.921
18	.093	.298	55	.417	.678	85	.737	.928
19	.101	.310						
20	.109	.321	56	.427	.688	86	.749	.935
			57	.437	.696	87	.760	.942
21	.116	.333	58	.447	.706	88	.773	.949
22	.124	.344	59	.457	.714	89	.786	.956
23	.132	.356	60	.466	.724	90	.798	.962
24	.140	.367						
25	.148	.378	61	.477	.732	91	.811	.968
			62	.486	.741	92	.824	.974
26	.156	.388	63	.497	.751	93	.837	.979
27	.164	399	64	.507	.760	94	.851	.984
28	.172	.409	65	.518	.768	95	.865	.989
29	.180	.419						
30	.189	.431				96	.880	.993
						97	.895	.997
31	.197	.441				98	.911	.999
32	.206	.452				99	.928	1.000
33	.214	.462				100	.948	1.000
34	.223	.473						
35	.232	.482						

Example: Observed from sample 50/100. The 99% confidence limits for the population are .369 and .631.

Table 3

CONFIDENCE IN INFERRING (90% ≤ p) FOR BINOMIAL DISTRIBUTION

The following tables list the confidence value in the body of the table in inferring that 90% < p for a binomial distribution.

These tables are useful for answering such questions as "if (x) units out of a sample of size (n) are observed to have some particular attribute, what confidence can be put in the statement that the true proportion of the population having this attribute is greater than 90%."

If the above question is asked about many different situations, then the table entry lists the percentage of situations in which p is actually greater than 90%.

Thus, the tables list for each sample size, n, and each observed number, x, a value for P such that

$$P(90\% \leq p) = \text{table entry}$$

Examples are given on each table.

PERCENTAGE CONFIDENCE IN INFERRING 90% ≤ n ≤ 100%

x \ n	1	2	3	4	5	6	7	8	9	10
0	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
1	10	1	<1	<1	<1	<1	<1	<1	<1	<1
2		19	3	<1	<1	<1	<1	<1	<1	<1
3			27	5	.1	<1	<1	<1	<1	<1
4				34	8	2	<1	<1	<1	<1
5					41	11	3	<1	<1	<1
6						47	15	4	<1	<1
7							52	19	5	1
8								57	23	7
9									61	26
10										65

Table 3 (Continued)

x \ n	11	12	13	14	15	16	17	18	19	20
7	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
8	2	<1	<1	<1	<1	<1	<1	<1	<1	<1
9	9	3	<1	<1	<1	<1	<1	<1	<1	<1
10	30	11	3	<1	<1	<1	<1	<1	<1	<1
11	69	34	13	4	1	<1	<1	<1	<1	<1
12		72	38	16	6	2	<1	<1	<1	<1
13			75	42	18	7	2	<1	<1	<1
14				77	45	21	8	3	<1	<1
15					79	49	24	10	4	1
16						81	52	27	11	4
17							83	55	29	13
18								85	58	32
19									86	61
20										88

Example: Observed from sample 16/17
          Confidence in inferring $90\% \leq p \leq 100\%$ is 52%

Table 3 (Continued)

x \ n	21	22	23	24	25	26	27	28	29	30
≤15	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
16	1	<1	<1	<1	<1	<1	<1	<1	<1	<1
17	5	2	<1	<1	<1	<1	<1	<1	<1	<1
18	15	6	2	<1	<1	<1	<1	<1	<1	<1
19	35	17	7	3	<1	<1	<1	<1	<1	<1
20	64	38	19	9	3	1	<1	<1	<1	<1
21	89	66	41	21	10	4	1	<1	<1	<1
22		90	68	44	24	11	5	2	<1	<1
23			91	71	46	26	13	6	2	<1
24				92	73	49	28	14	6	3
25					93	75	52	31	16	7
26						94	77	54	33	18
27							94	78	57	35
28								95	80	59
29									95	82
30										96

Example: Observed from sample 21/24
Confidence in inferring 90% ≤ p ≤ 100% is 21%

Table 3 (Continued)

x \ n	31	32	33	34	35	36	37	38	39	40
≤23	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
24	1	<1	<1	<1	<1	<1	<1	<1	<1	<1
25	3	1	<1	<1	<1	<1	<1	<1	<1	<1
26	8	4	1	<1	<1	<1	<1	<1	<1	<1
27	19	9	4	2	1	<1	<1	<1	<1	<1
28	38	21	11	5	2	<1	<1	<1	<1	<1
29	61	40	23	12	6	2	<1	<1	<1	<1
30	83	63	42	25	13	6	3	1	<1	<1
31	96	84	65	45	27	15	7	3	1	<1
32		97	86	67	47	29	16	8	4	2
33			97	87	69	49	31	17	9	4
34				97	88	71	51	33	19	10
35					97	89	73	54	35	21
36						98	90	75	56	37
37							98	90	76	58
38								98	91	78
39									98	92
40										99

Example: Observed from sample 34/36
Confidence in inferring 90% ≤ p ≤ 100% is 71%

Table 3 (Continued)

x \ n	45	50
≤ 35	<1	<1
36	1	<1
37	3	<1
38	8	<1
39	16	<1
40	29	2
41	47	6
42	67	12
43	84	23
44	95	33
45	99	38
46		57
47		75
48		89
49		97
50		>99

x \ n	55	60
≤44	<1	<1
45	2	<1
46	4	<1
47	9	<1
48	18	<1
49	31	1
50	48	3
51	65	7
52	81	14
53	92	25
54	98	39
55	>99	56
56		73
57		86
58		94
59		99
60		>99

x \ n	65	70
≤52	<1	<1
53	1	<1
54	3	<1
55	6	<1
56	11	<1
57	20	<1
58	32	2
59	48	4
60	64	9
61	79	16
62	90	26
63	96	40
64	99	56
65	>99	71
66		84
67		93
68		98
69		>99
70		>99

Table 3 (Continued)

x \ n	75	80
≤61	<1	<1
62	2	<1
63	4	<1
64	7	<1
65	13	<1
66	21	1
67	33	3
68	48	5
69	63	10
70	77	17
71	88	28
72	95	41
73	98	55
74	>99	70
75	>99	82
76		91
77		96
78		99
79		>99
80		>99

x \ n	85	90
≤69	<1	<1
70	1	<1
71	2	<1
72	4	<1
73	8	<1
74	14	<1
75	23	2
76	34	3
77	48	6
78	62	11
79	76	19
80	86	29
81	94	41
82	98	55
83	99	69
84	>99	81
85	>99	90
86		95
87		98
88		>99
89		>99
90		>99

x \ n	95	100
≤78	<1	<1
79	1	<1
80	3	<1
81	5	<1
82	9	<1
83	15	1
84	24	2
85	35	4
86	48	7
87	62	12
88	75	20
89	85	30
90	92	42
91	97	55
92	99	68
93	>99	79
94	>99	88
95	>99	94
96		98
97		99
98		>99
99		>99
100		>99

Example: Observed from sample 76/85
Confidence in inferring 90% ≤ p ≤ 100% is 34%

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Table 3 (Continued)

CONFIDENCE IN INFERRING (95% ≤ p) FOR BINOMIAL DISTRIBUTION

The following tables list the confidence value in the body of the table in inferring that 95% < p for a binomial distribution.

These tables are useful for answering such questions as "if (x) units out of a sample of size (n) are observed to have some particular attribute, what confidence can be put in the statement that the true proportion of the population having this attribute is greater than 95%."

If the above question is asked about many different situations, then the table entry lists the percentage of situations in which p is actually greater than 95%.

Thus, the tables list for each sample size, n, and each observed number, x, a value for P such that

$$P(95\% \leq p) = \text{table entry}$$

Examples are given on each table.

x \ n	1	2	3	4	5	6	7	8	9	10
0	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
1	5	<1	<1	<1	<1	<1	<1	<1	<1	<1
2		10	1	<1	<1	<1	<1	<1	<1	<1
3			14	1	<1	<1	<1	<1	<1	<1
4				19	2	<1	<1	<1	<1	<1
5					23	3	<1	<1	<1	<1
6						26	4	1	<1	<1
7							30	6	1	<1
8								34	7	1
9									37	9
10										40

Table 3 (Continued)

x \ n	11	12	13	14	15	16	17	18	19	20
≤8	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
9	2	<1	<1	<1	<1	<1	<1	<1	<1	<1
10	10	2	<1	<1	<1	<1	<1	<1	<1	<1
11	43	12	2	<1	<1	<1	<1	<1	<1	<1
12		46	14	3	1	<1	<1	<1	<1	<1
13			49	15	4	1	<1	<1	<1	<1
14				51	17	4	1	<1	<1	<1
15					54	19	5	1	<1	<1
16						56	21	6	1	<1
17							58	23	7	2
18								60	25	8
19									62	26
20										64

Example: Observed from sample 14/15
Confidence in inferring 95% ≤ p ≤ 100% is 17%

x \ n	21	22	23	24	25	26	27	28	29	30
≤17	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
18	2	<1	<1	<1	<1	<1	<1	<1	<1	<1
19	8	2	<1	<1	<1	<1	<1	<1	<1	<1
20	28	9	3	1	<1	<1	<1	<1	<1	<1
21	66	30	11	3	1	<1	<1	<1	<1	<1
22		67	32	12	3	<1	<1	<1	<1	<1
23			69	34	13	4	1	<1	<1	<1
24				71	36	14	4	1	<1	<1
25					72	38	15	5	1	<1
26						74	39	16	5	2
27							75	41	18	6
28								76	43	19
29									77	45
30										79

Table 3 (Continued)

x \ n	75	80
≤66	<1	<1
67	1	<1
68	3	<1
69	8	<1
70	17	<1
71	32	<1
72	52	1
73	73	4
74	89	10
75	98	21
76		37
77		57
78		77
79		91
80		98

x \ n	85	90
≤76	<1	<1
77	3	<1
78	6	<1
79	13	<1
80	25	<1
81	42	1
82	62	4
83	80	8
84	93	16
85	99	29
86		47
87		66
88		83
89		94
90		99

x \ n	95	100
≤85	<1	<1
86	2	<1
87	5	<1
88	10	<1
89	20	<1
90	34	1
91	52	3
92	71	6
93	86	13
94	95	23
95	99	38
96		56
97		74
98		88
99		96
100		99

Example: Observed from sample 90/90
Confidence in inferring 95% $\leq p \leq$ 100% is 99%

Table 3 (Continued)

x \ n	75	80
≤66	<1	<1
67	1	<1
68	3	<1
69	8	<1
70	17	<1
71	32	<1
72	52	1
73	73	4
74	89	10
75	98	21
76		37
77		57
78		77
79		91
80		98

x \ n	85	90
≤76	<1	<1
77	3	<1
78	6	<1
79	13	<1
80	25	<1
81	42	1
82	62	4
83	80	8
34	93	16
85	99	29
86		47
87		66
88		83
89		94
90		99

x \ n	95	100
≤85	<1	<1
86	2	<1
87	5	<1
88	10	<1
89	20	<1
90	34	1
91	52	3
92	71	6
93	86	13
94	95	23
95	99	38
96		56
97		74
98		88
99		96
100		99

Example: Observed from sample 90/90
Confidence in inferring 95% ≤ p ≤ 100% is 99%

Table 3 (Continued)

CONFIDENCE IN INFERRING (97% ≤ p) FOR BINOMIAL DISTRIBUTION

The following tables list the confidence value in the body of the table in inferring that 97% < p for a binomial distribution.

These tables are useful for answering such questions as "if x units out of a sample of size n are observed to have some particular attribute, what confidence can be put in the statement that the true proportion of the population having this attribute is greater than 97%."

If the above question is asked about many different situations, then the table entry lists the percentage of situations in which p is actually greater the 97%.

Thus, the tables list for each sample size, n, and each observed number, x, a value for P such that

$$P\ (97\% \le p) = \text{table entry}.$$

Examples are given on each table.

x \ n	1	2	3	4	5	6	7	8	9	10
0	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
1	3	<1	<1	<1	<1	<1	<1	<1	<1	<1
2		6	<1	<1	<1	<1	<1	<1	<1	<1
3			9	<1	<1	<1	<1	<1	<1	<1
4				11	<1	<1	<1	<1	<1	<1
5					14	<1	<1	<1	<1	<1
6						17	2	<1	<1	<1
7							19	2	<1	<1
8								22	3	<1
9									24	3
10										26

Table 3 (Continued)

x \ n	11	12	13	14	15	16	17	18	19	20
≤9	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
10	4	<1	<1	<1	<1	<1	<1	<1	<1	<1
11	28	5	<1	<1	<1	<1	<1	<1	<1	<1
12		32	6	<1	<1	<1	<1	<1	<1	<1
13			33	6	<1	<1	<1	<1	<1	<1
14				35	7	<1	<1	<1	<1	<1
15					37	8	1	<1	<1	<1
16						39	9	2	<1	<1
17							40	10	2	<1
18								42	11	2
19									44	12
20										46

Example: Observed from sample 19/20
Confidence in inferring 97% ≤ p ≤ 100% is 12%

x \ n	21	22	23	24	25	26	27	28	29	30
≤18	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
19	2	<1	<1	<1	<1	<1	<1	<1	<1	<1
20	13	3	<1	<1	<1	<1	<1	<1	<1	<1
21	47	14	3	<1	<1	<1	<1	<1	<1	<1
22		49	15	3	<1	<1	<1	<1	<1	<1
23			50	16	4	<1	<1	<1	<1	<1
24				52	17	4	<1	<1	<1	<1
25					53	18	5	<1	<1	<1
26						55	19	5	1	<1
27							56	20	6	1
28								57	22	6
29									59	23
30										60

Table 3 (Continued)

x \ n	31	32	33	34	35	36	37	38	39	40
≤27	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
28	1	<1	<1	<1	<1	<1	<1	<1	<1	<1
29	7	1	<1	<1	<1	<1	<1	<1	<1	<1
30	24	7	2	<1	<1	<1	<1	<1	<1	<1
31	61	25	8	2	<1	<1	<1	<1	<1	<1
32		62	26	8	2	<1	<1	<1	<1	<1
33			63	27	9	<1	<1	<1	<1	<1
34				64	28	9	2	<1	<1	<1
35					66	29	10	3	<1	<1
36						67	31	11	3	<1
37							68	32	11	3
38								69	33	12
39									70	34
40										70

Example: Observed from sample 35/36
Confidence in inferring 97% ≤ p ≤ 100% is 29%

x \ n	45	50
≤40	<1	<1
41	1	<1
42	5	<1
43	15	<1
44	39	<1
45	75	<1
46		2
47		6
48		19
49		44
50		78

x \ n	55	60
≤50	<1	<1
51	2	<1
52	8	<1
53	23	<1
54	49	<1
55	81	<1
56		3
57		11
58		27
59		54
60		84

x \ n	65	70
≤59	<1	<1
60	1	<1
61	5	<1
62	13	<1
63	31	<1
64	58	<1
65	86	2
66		6
67		16
68		35
69		62
70		88

Table 3 (Continued)

x \ n	75	80
≤69	<1	<1
70	3	<1
71	8	<1
72	19	<1
73	39	<1
74	66	1
75	90	3
76		9
77		22
78		43
79		70
80		91

x \ n	85	90
≤78	<1	<1
79	1	<1
80	4	<1
81	11	<1
82	25	<1
83	47	<1
84	73	2
85	92	5
86		13
87		28
88		51
89		76
90		94

x \ n	95	100
≤88	<1	<1
89	2	<1
90	7	<1
91	16	<1
92	32	<1
93	55	1
94	78	3
95	94	8
96		18
97		35
98		58
99		81
100		95

Example: Observed from sample 93/95
Confidence in inferring 97% ≤ p ≤ 100% is 55%

APPENDIX 3B

Table 3 (Continued)

CONFIDENCE IN INFERRING (99% < p) FOR BINOMIAL DISTRIBUTION

The following tables list the confidence value in the body of the table in inferring that 99% < p for a binomial distribution.

These tables are useful for answering such questions as "if (x) units out of a sample of size (n) are observed to have some particular attribute, what confidence can be put in the statement that the true proportion of the population having this attribute is greater than 99%."

If the above question is asked about many different situations, then the table entry lists the percentage of situations in which p is acutally greater than 99%.

Thus, the tables list for each sample size, n, and each observed number, x, a value for P such that

$$P \ (99\% \leq p) = \text{table entry}$$

Examples are given on each table.

x \ n	1	2	3	4	5	6	7	8	9	10
0	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
1	1	<1	<1	<1	<1	<1	<1	<1	<1	<1
2		2	<1	<1	<1	<1	<1	<1	<1	<1
3			3	<1	<1	<1	<1	<1	<1	<1
4				4	<1	<1	<1	<1	<1	<1
5					5	<1	<1	<1	<1	<1
6						6	<1	<1	<1	<1
7							7	<1	<1	<1
8								8	<1	<1
9									9	<1
10										10

Table 3 (Continued)

x \ n	11	12	13	14	15	16	17	18	19	20
≤10	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
11	10	<1	<1	<1	<1	<1	<1	<1	<1	<1
12		11	<1	<1	<1	<1	<1	<1	<1	<1
13			12	<1	<1	<1	<1	<1	<1	<1
14				13	1	<1	<1	<1	<1	<1
15					14	1	<1	<1	<1	<1
16						15	1	<1	<1	<1
17							16	1	<1	<1
18								17	2	<1
19									17	2
20										18

Example: Observed from sample 16/16
Confidence in inferring 99% ≤ p ≤ 100% is 15%

x \ n	21	22	23	24	25	26	27	28	29	30
≤19	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
20	2	<1	<1	<1	<1	<1	<1	<1	<1	<1
21	19	2	<1	<1	<1	<1	<1	<1	<1	<1
22		20	2	<1	<1	<1	<1	<1	<1	<1
23			21	2	<1	<1	<1	<1	<1	<1
24				21	3	<1	<1	<1	<1	<1
25					22	3	<1	<1	<1	<1
26						23	3	<1	<1	<1
27							24	3	<1	<1
28								24	3	<1
29									25	4
30										26

Table 3 (Continued)

x \ n	31	32	33	34	35	36	37	38	39	40
≤29	<1	<1	<1	<1	<1	<1	<1	<1	<1	<1
30	4	<1	<1	<1	<1	<1	<1	<1	<1	<1
31	27	4	<1	<1	<1	<1	<1	<1	<1	<1
32		28	4	<1	<1	<1	<1	<1	<1	<1
33			28	5	<1	<1	<1	<1	<1	<1
34				29	5	<1	<1	<1	<1	<1
35					30	5	<1	<1	<1	<1
36						30	5	<1	<1	<1
37							31	6	<1	<1
38								32	6	<1
39									32	6
40										33

Example: Observed from sample 27/ 28
Confidence in inferring 99% $\leq p \leq$ 100% is 3%

x \ n	45	50
≤42	<1	<1
43	1	<1
44	7	<1
45	36	<1
46		<1
47		<1
48		1
49		9
50		39

x \ n	55	60
≤52	<1	<1
53	2	<1
54	11	<1
55	42	<1
56		<1
57		<1
58		2
59		12
60		45

x \ n	65	70
≤62	<1	<1
63	3	<1
64	14	<1
65	48	<1
66		<1
67		<1
68		3
69		16
70		51

x \ n	75	80
≤72	<1	<1
73	4	<1
74	17	<1
75	53	<1
76		<1
77		<1
78		5
79		19
80		55

Table 3 (Continued)

x \ n	85	90
≤81	<1	<1
82	1	<1
83	5	<1
84	21	<1
85	57	<1
86		<1
87		1
88		6
89		23
90		60

x \ n	95	100
≤91	<1	<1
92	2	<1
93	7	<1
94	25	<1
95	62	<1
96		<1
97		2
98		8
99		26
100		63

Example: Observed from sample 74/75
Confidence in inferring $97\% \leq p \leq 100\%$ is 17%
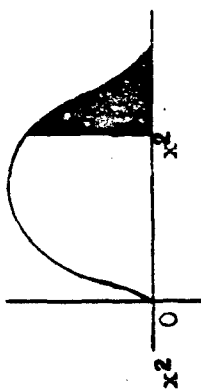
APPENDIX 3C

CHI-SQUARE DISTRIBUTION

Degrees of Freedom	P = 0.99	0.98	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01
1	0.000157	0.000628	0.000393	0.0158	0.0642	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635
2	0.0201	0.0404	0.103	0.211	0.446	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210
3	0.115	0.185	0.352	0.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341
4	0.297	0.429	0.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086
6	0.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638
24	11.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892

APPENDIX 3D

Table 1

CONFIDENCE LIMITS FOR THE EXPECTATION OF A POISSON VARIABLE

1-2α	0.998		0.99		0.98		0.95		0.90		1-2α
α	0.001		0.005		0.01		0.025		0.05		α
c	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	c
0	0.00000	6.91	0.00000	5.30	0.0000	4.61	0.0000	3.69	0.0000	3.00	0
1	.00100	9.23	.00501	7.43	.0101	6.64	0.253	5.57	.0513	4.74	1
2	.0454	11.23	.103	9.27	.149	8.41	.242	7.22	.355	6.30	2
3	.191	13.06	.338	10.98	.436	10.05	.619	8.77	.818	7.75	3
4	.429	14.79	.672	12.59	.823	11.60	1.09	10.24	1.37	9.15	4
5	0.739	16.45	1.08	14.15	1.28	13.11	1.62	11.67	1.97	10.51	5
6	1.11	18.06	1.54	15.66	1.79	14.57	2.20	13.06	2.61	11.84	6
7	1.52	19.63	2.04	17.13	2.33	16.00	2.81	14.42	3.29	13.15	7
8	1.97	21.16	2.57	18.58	2.91	17.40	3.45	15.76	3.98	14.43	8
9	2.45	22.66	3.13	20.00	3.51	18.78	4.12	17.08	4.70	15.71	9
10	2.96	24.13	3.72	21.40	4.13	20.14	4.80	18.39	5.43	16.96	10
11	3.49	25.59	4.32	22.78	4.77	21.49	5.49	19.68	6.17	18.21	11
12	4.04	27.03	4.94	24.14	5.43	22.82	6.20	20.96	6.92	19.44	12
13	4.61	28.45	5.58	25.50	6.10	24.14	6.92	22.23	7.69	20.67	13
14	5.20	29.85	6.23	26.84	6.78	25.45	7.65	23.49	8.46	21.89	14
15	5.79	31.24	6.89	28.16	7.48	26.74	8.40	24.74	9.25	23.10	15
16	6.41	32.62	7.57	29.48	8.18	28.03	9.15	25.98	10.04	24.30	16
17	7.03	33.99	8.25	30.79	8.89	29.31	9.90	27.22	10.83	25.50	17
18	7.66	35.35	8.94	32.09	9.62	30.58	10.67	28.45	11.63	26.69	18
19	8.31	36.70	9.64	33.38	10.35	31.85	11.44	29.67	12.44	27.88	19
20	8.96	38.04	10.35	34.67	11.08	33.10	12.22	30.89	13.25	29.06	20
21	9.62	39.38	11.07	35.95	11.82	34.36	13.00	32.10	14.07	30.24	21
22	10.29	40.70	11.79	37.22	12.57	35.60	13.79	33.31	14.89	31.42	22
23	10.96	42.02	12.52	38.48	13.33	36.84	14.58	34.51	15.72	32.59	23
24	11.65	43.33	13.25	39.74	14.09	38.08	15.38	35.71	16.55	33.75	24
25	12.34	44.64	14.00	41.00	14.85	39.31	16.18	36.90	17.38	34.92	25
26	13.03	45.94	14.74	42.25	15.62	40.53	16.98	38.10	18.22	36.08	26
27	13.73	47.23	15.49	43.50	16.40	41.76	17.79	39.28	19.06	37.23	27
28	14.44	48.52	16.24	44.74	17.17	42.98	18.61	40.47	19.90	38.39	28
29	15.15	49.80	17.00	45.98	17.96	44.19	19.42	41.65	20.75	39.54	29
30	15.87	51.08	17.77	47.21	18.74	45.40	20.24	42.83	21.59	40.69	30
35	19.52	57.42	21.64	53.32	22.72	51.41	24.38	48.68	25.87	46.40	35
40	23.26	63.66	25.59	59.36	26.77	57.35	28.58	54.47	30.20	52.07	40
45	27.08	69.83	29.60	65.34	30.88	63.23	32.82	60.21	34.56	57.69	45
50	30.96	75.94	33.66	71.27	35.03	69.07	37.11	65.92	38.96	63.29	50

If c is the observed frequency or count and m_A, m_B are the lower and upper confidence limits for its expectation, m, then

$$Pr(m_A \leq m \leq m_B) \leq 1-2\alpha$$

APPENDIX 3D

Table 1

CONFIDENCE LIMITS FOR THE EXPECTATION OF A POISSON VARIABLE

1-2α	0.998		0.99		0.98		0.95		0.90		1-2α
α	0.001		0.005		0.01		0.025		0.05		α
c	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	c
0	0.00000	6.91	0.00000	5.30	0.0000	4.61	0.0000	3.69	0.0000	3.00	0
1	.00100	9.23	.00501	7.43	.0101	6.64	0.253	5.57	.0513	4.74	1
2	.0454	11.23	.103	9.27	.149	8.41	.242	7.22	.355	6.30	2
3	.191	13.06	.338	10.98	.436	10.05	.619	8.77	.818	7.75	3
4	.429	14.79	.672	12.59	.823	11.60	1.09	10.24	1.37	9.15	4
5	0.739	16.45	1.08	14.15	1.28	13.11	1.62	11.67	1.97	10.51	5
6	1.11	18.06	1.54	15.66	1.79	14.57	2.20	13.06	2.61	11.84	6
7	1.52	19.63	2.04	17.15	2.33	16.00	2.81	14.42	3.29	13.15	7
8	1.97	21.16	2.57	18.58	2.91	17.40	3.45	15.76	3.98	14.43	8
9	2.45	22.66	3.13	20.00	3.51	18.78	4.12	17.08	4.70	15.71	9
10	2.96	24.13	3.72	21.40	4.13	20.14	4.80	18.39	5.43	16.96	10
11	3.49	25.59	4.32	22.78	4.77	21.49	5.49	19.68	6.17	18.21	11
12	4.04	27.03	4.94	24.14	5.43	22.82	6.20	20.96	6.92	19.44	12
13	4.61	28.45	5.58	25.50	6.10	24.14	6.92	22.23	7.69	20.67	13
14	5.20	29.85	6.23	26.84	6.78	25.45	7.65	23.49	8.46	21.89	14
15	5.79	31.24	6.89	28.16	7.48	26.74	8.40	24.74	9.25	23.10	15
16	6.41	32.62	7.57	29.48	8.18	28.03	9.15	25.98	10.04	24.30	16
17	7.03	33.99	8.25	30.79	8.89	29.31	9.90	27.22	10.83	25.50	17
18	7.66	35.35	8.94	32.09	9.62	30.58	10.67	28.45	11.63	26.69	18
19	8.31	36.70	9.64	33.38	10.35	31.85	11.44	29.67	12.44	27.88	19
20	8.96	38.04	10.35	34.67	11.08	33.10	12.22	30.89	13.25	29.06	20
21	9.62	39.38	11.07	35.95	11.82	34.36	13.00	32.10	14.07	30.24	21
22	10.29	40.70	11.79	37.22	12.57	35.60	13.79	33.31	14.89	31.42	22
23	10.96	42.02	12.52	38.48	13.33	36.84	14.58	34.51	15.72	32.59	23
24	11.65	43.33	13.25	39.74	14.09	38.08	15.38	35.71	16.55	33.75	24
25	12.34	44.64	14.00	41.00	14.85	39.31	16.18	36.90	17.38	34.92	25
26	13.03	45.94	14.74	42.25	15.62	40.53	16.98	38.10	18.22	36.08	26
27	13.73	47.23	15.49	43.50	16.40	41.76	17.79	39.28	19.06	37.23	27
28	14.44	48.52	16.24	44.74	17.17	42.93	18.61	40.47	19.90	38.39	28
29	15.15	49.80	17.00	45.98	17.96	44.19	19.42	41.65	20.75	39.54	29
30	15.87	51.08	17.77	47.21	18.74	45.40	20.24	42.83	21.59	40.69	30
35	19.52	57.42	21.64	53.32	22.72	51.41	24.38	48.68	25.87	46.40	35
40	23.26	63.66	25.59	59.36	26.77	57.35	28.58	54.47	30.20	52.07	40
45	27.08	69.83	29.60	65.34	30.88	63.23	32.82	60.21	34.56	57.69	45
50	30.96	75.94	33.66	71.27	35.53	69.07	37.11	65.92	38.96	63.29	50

If c is the observed frequency or count and m_A, m_B are the lower and upper confidence limits for its expectation, m, then

$$Pr(m_A \leq m \leq m_B) \leq 1-2\alpha$$

Table 1A

F DISTRIBUTION: UPPER 10 PERCENT POINTS

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	39.864	49.500	53.593	55.833	57.241	58.204	58.906	59.439	59.858
2	8.5263	9.0000	9.1618	9.2434	9.2926	9.3255	9.3491	9.3668	9.3805
3	5.5383	5.4624	5.3908	5.3427	5.3092	5.2847	5.2662	5.2517	5.2400
4	4.5448	4.3246	4.1908	4.1073	4.0506	4.0098	3.9790	3.9549	3.9357
5	4.0604	3.7797	3.6195	3.5202	3.4530	3.4045	3.3679	3.3393	3.3163
6	3.7760	3.4633	3.2888	3.1808	3.1075	3.0546	3.0145	2.9830	2.9577
7	3.5894	3.2574	3.0741	2.9605	2.8833	2.8274	2.7849	2.7516	2.7247
8	3.4579	3.1131	2.9238	2.8064	2.7265	2.6683	2.6241	2.5893	2.5612
9	3.3603	3.0065	2.8129	2.6927	2.6106	2.5509	2.5053	2.4694	2.4403
10	3.2850	2.9245	2.7277	2.6053	2.5216	2.4606	2.4140	2.3772	2.3473
11	3.2252	2.8595	2.6602	2.5362	2.4512	2.3891	2.3416	2.3040	2.2735
12	3.1765	2.8068	2.6055	2.4801	2.3940	2.3310	2.2828	2.2446	2.2135
13	3.1362	2.7632	2.5603	2.4337	2.3467	2.2830	2.2341	2.1953	2.1638
14	3.1022	2.7265	2.5222	2.3947	2.3069	2.2426	2.1931	2.1539	2.1220
15	3.0732	2.6952	2.4898	2.3614	2.2730	2.2081	2.1582	2.1185	2.0862
16	3.0481	2.6682	2.4618	2.3327	2.2438	2.1783	2.1280	2.0880	2.0553
17	3.0262	2.6446	2.4374	2.3077	2.2183	2.1524	2.1017	2.0613	2.0284
18	3.0070	2.6239	2.4160	2.2858	2.1958	2.1296	2.0785	2.0379	2.0047
19	2.9899	2.6056	2.3970	2.2663	2.1760	2.1094	2.0580	2.0171	1.9836
20	2.9747	2.5893	2.3801	2.2489	2.1582	2.0913	2.0397	1.9985	1.9649
21	2.9609	2.5746	2.3649	2.2333	2.1423	2.0751	2.0232	1.9819	1.9480
22	2.9486	2.5613	2.3512	2.2193	2.1279	2.0605	2.0084	1.9668	1.9327
23	2.9374	2.5493	2.3387	2.2065	2.1149	2.0472	1.9949	1.9531	1.9189
24	2.9271	2.5383	2.3274	2.1949	2.1030	2.0351	1.9826	1.9407	1.9063
25	2.9177	2.5283	2.3170	2.1843	2.0922	2.0241	1.9714	1.9292	1.8947
26	2.9091	2.5191	2.3075	2.1745	2.0822	2.0139	1.9610	1.9188	1.8841
27	2.9012	2.5106	2.2987	2.1655	2.0730	2.0045	1.9515	1.9091	1.8743
28	2.8939	2.5028	2.2906	2.1571	2.0645	1.9959	1.9427	1.9001	1.8652
29	2.8871	2.4955	2.2831	2.1494	2.0566	1.9878	1.9345	1.8918	1.8568
30	2.8807	2.4887	2.2761	2.1422	2.0492	1.9803	1.9269	1.8841	1.8490
40	2.8354	2.4404	2.2261	2.0909	1.9968	1.9269	1.8725	1.8289	1.7929
60	2.7914	2.3932	2.1774	2.0410	1.9457	1.8747	1.8194	1.7748	1.7380
120	2.7478	2.3473	2.1300	1.9923	1.8959	1.8238	1.7675	1.7220	1.6843
∞	2.7055	2.3026	2.0838	1.9449	1.8473	1.7741	1.7167	1.6702	1.6315

This table gives the values of F for which $I_F(v_1, v_2) = 0.10$

One-sided 90 percent test.

Two-sided 80 percent test.

APPENDIX 3E

Table 1B

F DISTRIBUTION: UPPER 10 PERCENT POINTS

V_2 \ V_1	10	12	15	20	24	30	40	60	120	∞
1	60.195	60.705	61.220	61.740	62.002	62.265	62.529	62.794	63.061	63.328
2	9.3916	9.4081	9.4247	9.4413	9.4496	9.4579	9.4663	9.4746	9.4829	9.4913
3	5.2304	5.2156	5.2003	5.1845	5.1764	5.1681	5.1597	5.1512	5.1425	5.1337
4	3.9199	3.8955	3.8689	3.8443	3.8310	3.8174	3.8036	3.7896	3.7753	3.7607
5	3.2974	3.2682	3.2380	3.2067	3.1905	3.1741	3.1573	3.1402	3.1228	3.1050
6	2.9369	2.9047	2.8712	2.8363	2.8183	2.8000	2.7812	2.7620	2.7423	2.7222
7	2.7025	2.6681	2.6322	2.5947	2.5753	2.5555	2.5351	2.5142	2.4928	2.4708
8	2.5380	2.5020	2.4642	2.4246	2.4041	2.3830	2.3614	2.3391	2.3162	2.2926
9	2.4163	2.3789	2.3396	2.2983	2.2768	2.2547	2.2320	2.2085	2.1843	2.1592
10	2.3226	2.2841	2.2435	2.2007	2.1784	2.1554	2.1317	2.1072	2.0818	2.0554
11	2.2482	2.2087	2.1671	2.1230	2.1000	2.0762	2.0516	2.0261	1.9997	1.9721
12	2.1878	2.1474	2.1049	2.0597	2.0360	2.0115	1.9861	1.9597	1.9323	1.9036
13	2.1376	2.0966	2.0532	2.0070	1.9827	1.9576	1.9315	1.9043	1.8759	1.8462
14	2.0954	2.0537	2.0095	1.9625	1.9377	1.9119	1.8852	1.8572	1.8280	1.7973
15	2.0593	2.0171	1.9722	1.9243	1.8990	1.8728	1.8454	1.8168	1.7867	1.7551
16	2.0281	1.9854	1.9399	1.8913	1.8656	1.8388	1.8108	1.7816	1.7507	1.7182
17	2.0009	1.9577	1.9117	1.8624	1.8362	1.8090	1.7805	1.7506	1.7191	1.6856
18	1.9770	1.9333	1.8868	1.8368	1.8103	1.7827	1.7537	1.7232	1.6910	1.6567
19	1.9557	1.9117	1.8647	1.8142	1.7873	1.7592	1.7298	1.6988	1.6659	1.6308
20	1.9367	1.8924	1.8449	1.7938	1.7667	1.7382	1.7083	1.6768	1.6433	1.6074
21	1.9197	1.8750	1.8272	1.7756	1.7481	1.7193	1.6890	1.6569	1.6228	1.5862
22	1.9043	1.8593	1.8111	1.7590	1.7312	1.7021	1.6714	1.6389	1.6042	1.5668
23	1.8903	1.8450	1.7964	1.7439	1.7159	1.6864	1.6554	1.6224	1.5871	1.5490
24	1.8775	1.8319	1.7831	1.7302	1.7019	1.6721	1.6407	1.6073	1.5715	1.5327
25	1.8658	1.8200	1.7708	1.7175	1.6890	1.6589	1.6272	1.5934	1.5570	1.5176
26	1.8550	1.8090	1.7596	1.7059	1.6771	1.6468	1.6147	1.5805	1.5437	1.5036
27	1.8451	1.7989	1.7492	1.6951	1.6662	1.6356	1.6032	1.5686	1.5313	1.4906
28	1.8359	1.7895	1.7395	1.6852	1.6560	1.6252	1.5925	1.5575	1.5198	1.4784
29	1.8274	1.7808	1.7306	1.6759	1.6465	1.6155	1.5825	1.5472	1.5090	1.4670
30	1.8195	1.7727	1.7223	1.6673	1.6377	1.6065	1.5732	1.5376	1.4989	1.4564
40	1.7627	1.7146	1.6624	1.6052	1.5741	1.5411	1.5056	1.4672	1.4248	1.3769
60	1.7070	1.6574	1.6034	1.5435	1.5107	1.4755	1.4373	1.3952	1.3476	1.2915
120	1.6524	1.6012	1.5450	1.4821	1.4472	1.4094	1.3676	1.3203	1.2646	1.1926
∞	1.5987	1.5458	1.4871	1.4206	1.3832	1.3419	1.2951	1.2400	1.1686	1.0000

$$F = \frac{s_1^2}{s_2^2} = \frac{v_2 S_1}{v_1 S_2}$$

One-sided 90 percent test.

Two-sided 80 percent test.

Table 2A

F DISTRIBUTION: UPPER 5 PERCENT POINTS

V_1 / V_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.5521	9.2766	9.1172	9.0135	.9406	8.8868	8.8452	8.8123
4	7.7086	6.9443	6.5914	6.3883	6.2560	6.1631	6.0942	6.0410	5.9988
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2066	4.1468	4.0990
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8	5.3177	4.4590	4.0662	3.8378	3.6875	3.5806	3.5005	3.4381	3.3881
9	5.1174	4.2565	3.8626	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6937	2.6143	2.5480	2.4943
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19	4.3803	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20	4.3513	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3661
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26	4.2252	3.3690	2.9751	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2782	2.2229
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40	4.0848	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2540	2.1665	2.0970	2.0401
120	3.9201	3.0718	2.6802	2.4472	2.2900	2.1750	2.0867	2.0164	1.9588
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

This table gives the values of F for which $I_F(V_1, V_2) = 0.05$.

One-sided 95 percent test.

Two-sided 90 percent test.

Table 2B

F DISTRIBUTION: UPPER 5 PERCENT POINTS

v_2 \ v_1	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.09	251.14	252.20	253.25	254.32
2	19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5265
4	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6878	5.6581	5.6281
5	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3984	4.3650
6	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6688
7	3.6365	3.5747	3.5108	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	3.3472	3.2840	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7473	2.7067
10	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	2.6021	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2230	2.1778	2.1307
15	2.5437	2.4753	2.4035	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9796	1.9302	1.8780
20	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8895	1.8380	1.7831
23	2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8649	1.8128	1.7570
24	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7897	1.7331
25	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7307	1.6717
28	2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6377
30	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
∞	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000

$$F = \frac{s_1^2}{s_2^2} = \frac{v_2}{v_1}\frac{S_1}{S_2}$$

One-sided 95 percent test.

Two-sided 90 percent test.

Table 3A

F DISTRIBUTION: UPPER 2.5 PERCENT POINTS

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
2	38.506	39.000	39.165	39.248	39.298	39.331	39.355	39.373	39.387
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.540	14.473
4	12.2'8	10.649	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047
5	7	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6810
6	8.0131	7.2598	6.5988	6.2272	5.9876	5.8197	5.6955	5.5996	5.5234
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8994	4.8232
8	7.5709	6.0595	5.4160	5.0526	4.8173	4.6517	4.5286	4.4332	4.3572
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.1971	4.1020	4.0260
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.7790
11	6.7241	5.2559	4.6300	4.2751	4.0440	3.8807	3.7586	3.6638	3.5879
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.3880	3.3120
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093
15	6.1995	4.7650	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488
17	6.0420	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.0610	2.9849
18	5.9781	4.5597	3.9539	3.6083	3.3820	3.2209	3.0999	3.0053	2.9291
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8800
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.8740	2.7977
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9024	2.8077	2.7313
24	5.7167	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027
25	5.6864	4.2909	3.6943	3.3530	3.1287	2.9685	2.8478	2.7531	2.6766
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.8240	2.7293	2.6528
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309
28	5.6096	4.2205	3.6264	3.2863	3.0625	2.9027	2.7820	2.6872	2.6106
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.8840	2.7633	2.6686	2.5919
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.7460	2.6513	2.5746
40	5.4239	4.0510	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519
60	5.2857	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.33
120	5.1524	3.8046	3.2270	2.8943	2.6740	2.5154	2.3948	2.2994	2.2217
∞	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136

This table gives the values of F for which $I_F(v_1, v_2) = 0.025$.

One-sided 97.5 percent test.
Two-sided 95.0 percent test.

Table 3B

F DISTRIBUTION: UPPER 2.5 PERCENT POINTS

V_2 \ V_1	10	12	15	20	24	30	40	60	120	∞
1	968.63	976.71	984.87	993.10	997.25	1001.4	1005.6	1009.8	1014.0	1018.3
2	39.398	39.415	39.431	39.448	39.456	39.465	39.473	39.481	39.490	39.498
3	14.419	14.337	14.253	14.167	14.124	14.081	14.037	13.992	13.947	13.902
4	8.8439	8.7512	8.6565	8.5599	8.5109	8.4613	8.4111	8.3604	8.3092	8.2573
5	6.6192	6.5246	6.4277	6.3285	6.2780	6.2269	6.1751	6.1225	6.0693	6.0153
6	5.4613	5.3662	5.2687	5.1684	5.1172	5.0652	5.0125	4.9589	4.9045	4.8491
7	4.7611	4.6658	4.5678	4.4667	4.4150	4.3624	4.3089	4.2544	4.1989	4.1423
8	4.2951	4.1997	4.1012	3.9995	3.9472	3.8940	3.8398	3.7844	3.7279	3.6702
9	3.9639	3.8682	3.7694	3.6669	3.6142	3.5604	3.5055	3.4493	3.3918	3.3329
10	3.7168	3.6209	3.5217	3.4186	3.3654	3.3110	3.2554	3.1984	3.1399	3.0798
11	3.5257	3.4296	3.3299	3.2261	3.1725	3.1176	3.0613	3.0035	2.9441	2.8828
12	3.3736	3.2773	3.1772	3.0728	3.0187	2.9633	2.9063	2.8478	2.7874	2.7249
13	3.2497	3.1532	3.0527	2.9477	2.8932	2.8373	2.7797	2.7204	2.6590	2.5955
14	3.1469	3.0501	2.9493	2.8437	2.7888	2.7324	2.6742	2.6142	2.5519	2.4872
15	3.0602	2.9633	2.8621	2.7559	2.7006	2.6437	2.5850	2.5242	2.4611	2.3953
16	2.9862	2.8890	2.7875	2.6808	2.6252	2.5678	2.5085	2.4471	2.3831	2.3163
17	2.9222	2.8249	2.7230	2.6158	2.5598	2.5021	2.4422	2.3801	2.3153	2.2474
18	2.8664	2.7689	2.6667	2.5590	2.5027	2.4445	2.3842	2.3214	2.2558	2.1869
19	2.8173	2.7196	2.6171	2.5089	2.4523	2.3937	2.3329	2.2695	2.2032	2.1333
20	2.7737	2.6758	2.5731	2.4645	2.4076	2.3486	2.2873	2.2234	2.1562	2.0853
21	2.7348	2.6368	2.5338	2.4247	2.3675	2.3082	2.2465	2.1819	2.1141	2.0422
22	2.6998	2.6017	2.4984	2.3890	2.3315	2.2718	2.2097	2.1446	2.0760	2.0032
23	2.6682	2.5699	2.4665	2.3567	2.2989	2.2389	2.1763	2.1107	2.0415	1.9677
24	2.6396	2.5412	2.4374	2.3273	2.2693	2.2090	2.1460	2.0799	2.0099	1.9353
25	2.6135	2.5149	2.4110	2.3005	2.2422	2.1816	2.1183	2.0517	1.9811	1.9055
26	2.5895	2.4909	2.3867	2.2759	2.2174	2.1565	2.0928	2.0257	1.9545	1.8781
27	2.5676	2.4688	2.3644	2.2533	2.1946	2.1334	2.0693	2.0018	1.9299	1.8527
28	2.5473	2.4484	2.3438	2.2324	2.1735	2.1121	2.0477	1.9796	1.9072	1.8291
29	2.5286	2.4295	2.3248	2.2131	2.1540	2.0923	2.0276	1.9591	1.8861	1.8072
30	2.5112	2.4120	2.3072	2.1952	2.1359	2.0739	2.0089	1.9400	1.8664	1.7867
40	2.3882	2.2882	2.1819	2.0677	2.0069	1.9429	1.8752	1.8028	1.7242	1.6371
60	2.2702	2.1692	2.0613	1.9445	1.8817	1.8152	1.7440	1.6668	1.5810	1.4822
120	2.1570	2.0548	1.9450	1.8249	1.7597	1.6899	1.6141	1.5299	1.4327	1.3104
∞	2.0483	1.9447	1.8326	1.7085	1.6402	1.5660	1.4835	1.3883	1.2684	1.0000

$$F = \frac{s_1^2}{s_2^2} = \frac{v_2 S_1}{v_1 S_2}$$

One-sided 97.5 percent test.
Two-sided 95.0 percent test.

Table 4A

F DISTRIBUTION: UPPER 1 PERCENT POINTS

v_2 \ v_1	1	2	3	4	5	6	7	8	9
1	4052.2	4999.5	5403.3	5624.6	5763.7	5859.0	5928.3	5981.6	6022.5
2	98.503	99.000	99.166	99.249	99.299	99.332	99.356	99.374	99.388
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6	13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1016	7.9761
7	12.246	9.5466	8.4513	7.8467	7.4604	7.1914	6.9928	6.8401	6.7188
8	11.259	8.6491	7.5910	7.0060	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9526	5.4119	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1850	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2635	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26	7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27	7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28	7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29	7.5976	5.4205	4.5378	4.0449	3.7254	3.4995	3.3302	3.1982	3.0920
30	7.5625	5.3904	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6491	3.3389	3.1187	2.9530	2.8233	2.7185
120	6.8510	4.7865	3.9493	3.4796	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

This table gives the values of F for which $I_F(v_1, v_2) = 0.01$.

One-sided 99 percent test.

Two-sided 98 percent test.

Table 4B

F DISTRIBUTION: UPPER 1 PERCENT POINTS

v_2 \ v_1	10	12	15	20	24	30	40	60	120	∞
1	6055.8	6106.3	6157.3	6208.7	6234.6	6260.7	6286.8	6313.0	6339.4	6366.0
2	99.399	99.416	99.432	99.449	99.458	99.466	99.474	99.483	99.491	99.501
3	27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4	14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5	10.051	9.8883	9.7222	9.5527	9.4665	9.3793	9.2912	9.2020	9.1118	9.0204
6	7.8741	7.7183	7.5590	7.3958	7.3127	7.2285	7.1432	7.0568	6.9690	6.8801
7	6.6201	6.4691	6.3143	6.1554	6.0743	5.9921	5.9084	5.8326	5.7572	5.6495
8	5.8143	5.6668	5.5151	5.3591	5.2793	5.1981	5.1156	5.0316	4.9460	4.8588
9	5.2565	5.1114	4.9621	4.8080	4.7290	4.6486	4.5667	4.4831	4.3978	4.3105
10	4.8492	4.7059	4.5582	4.4054	4.3269	4.2469	4.1653	4.0819	3.9965	3.9090
11	4.5393	4.3974	4.2509	4.0990	4.0209	3.9411	3.8596	3.7761	3.6904	3.6025
12	4.2961	4.1553	4.0096	3.8584	3.7805	3.7008	3.6192	3.5355	3.4494	3.3608
13	4.1003	3.9603	3.8154	3.6646	3.5868	3.5070	3.4253	3.3413	3.2548	3.1654
14	3.9394	3.8001	3.6557	3.5052	3.4274	3.3476	3.2656	3.1813	3.0942	3.0040
15	3.8049	3.6662	3.5222	3.3719	3.2940	3.2141	3.1319	3.0471	2.9595	2.8684
16	3.6909	3.5527	3.4089	3.2588	3.1808	3.1007	3.0182	2.9330	2.8447	2.7528
17	3.5931	3.4552	3.3117	3.1615	3.0835	3.0032	2.9205	2.8348	2.7451	2.6530
18	3.5082	3.3706	3.2273	3.0771	2.9990	2.9185	2.8354	2.7493	2.6597	2.5660
19	3.4338	3.2965	3.1533	3.0031	2.9249	2.8442	2.7608	2.6742	2.5839	2.4893
20	3.3682	3.2311	3.0880	2.9377	2.8594	2.7785	2.6947	2.6077	2.5168	2.4212
21	3.3098	3.1729	3.0299	2.8796	2.8011	2.7200	2.6359	2.5484	2.4568	2.3603
22	3.2576	3.1209	2.9780	2.8274	2.7488	2.6675	2.5831	2.4951	2.4029	2.3055
23	3.2106	3.0740	2.9311	2.7805	2.7017	2.6202	2.5355	2.4471	2.3542	2.2559
24	3.1681	3.0316	2.8887	2.7380	2.6591	2.5773	2.4923	2.4035	2.3099	2.2107
25	3.1294	2.9931	2.8502	2.6993	2.6203	2.5383	2.4530	2.3637	2.2695	2.1694
26	3.0941	2.9579	2.8150	2.6640	2.5848	2.5026	2.4170	2.3273	2.2325	2.1315
27	3.0618	2.9256	2.7827	2.6316	2.5522	2.4699	2.3840	2.2938	2.1984	2.0965
28	3.0320	2.8959	2.7530	2.6017	2.5223	2.4397	2.3535	2.2629	2.1670	2.0642
29	3.0045	2.8685	2.7256	2.5742	2.4946	2.4118	2.3253	2.2344	2.1378	2.0342
30	2.9791	2.8431	2.7002	2.5487	2.4689	2.3860	2.2992	2.2079	2.1107	2.0062
40	2.8005	2.6648	2.5216	2.3689	2.2880	2.2034	2.1142	2.0194	1.9172	1.8047
60	2.6318	2.4961	2.3523	2.1978	2.1154	2.0285	1.9360	1.8363	1.7263	1.6006
120	2.4721	2.3363	2.1915	2.0346	1.9500	1.8600	1.7628	1.6557	1.5330	1.3805
∞	2.3209	2.1848	2.0385	1.8783	1.7908	1.6964	1.5923	1.4730	1.3246	1.0000

$$F = \frac{s_1^2}{s_2^2} = \frac{v_2 \, S_1}{v_1 \, S_2}$$

One-sided 99 percent test.

Two-sided 98 percent test.

APPENDIX 3F

STUDENT'S t-DISTRIBUTION

df	.60	.70	.80	.90	.95	.975	.99	.995
1	.325	.727	1.376	3.078	6.314	12.706	31.821	63.657
2	.289	.617	1.061	1.886	2.920	4.303	6.965	9.925
3	.277	.584	.978	1.638	2.353	3.182	4.541	5.841
4	.271	.569	.941	1.533	2.132	2.776	3.747	4.604
5	.267	.559	.920	1.476	2.015	2.571	3.365	4.032
6	.265	.553	.906	1.440	1.943	2.447	3.143	3.707
7	.263	.549	.896	1.415	1.895	2.365	2.998	3.499
8	.262	.546	.889	1.397	1.860	2.306	2.896	3.355
9	.261	.543	.883	1.383	1.833	2.262	2.821	3.250
10	.260	.542	.879	1.372	1.812	2.228	2.764	3.169
11	.260	.540	.876	1.363	1.796	2.201	2.718	3.106
12	.259	.539	.873	1.356	1.782	2.179	2.681	3.055
13	.259	.538	.870	1.350	1.771	2.160	2.650	3.012
14	.258	.537	.868	1.345	1.761	2.145	2.624	2.977
15	.258	.536	.866	1.341	1.753	2.131	2.602	2.947
16	.258	.535	.865	1.337	1.746	2.120	2.583	2.921
17	.257	.534	.863	1.333	1.740	2.110	2.567	2.898
18	.257	.534	.862	1.330	1.734	2.101	2.552	2.878
19	.257	.533	.861	1.328	1.729	2.093	2.539	2.861
20	.257	.533	.860	1.325	1.725	2.086	2.528	2.845
21	.257	.532	.859	1.323	1.721	2.080	2.518	2.831
22	.256	.532	.858	1.321	1.717	2.074	2.508	2.819
23	.256	.532	.858	1.319	1.714	2.069	2.500	2.807
24	.256	.531	.857	1.318	1.711	2.064	2.492	2.797
25	.256	.531	.856	1.316	1.708	2.060	2.485	2.787
26	.256	.531	.856	1.315	1.706	2.056	2.479	2.779
27	.256	.531	.855	1.314	1.703	2.052	2.473	2.771
28	.256	.530	.855	1.313	1.701	2.048	2.467	2.763
29	.256	.530	.854	1.311	1.699	2.045	2.462	2.756
30	.256	.530	.854	1.310	1.697	2.042	2.457	2.750
40	.255	.529	.851	1.303	1.684	2.021	2.423	2.704
60	.254	.527	.848	1.296	1.671	2.000	2.390	2.660
120	.254	.526	.845	1.289	1.658	1.980	2.358	2.617
∞	.253	.524	.842	1.282	1.645	1.960	2.326	2.576
df								

APPENDIX 3G

AREAS UNDER THE STANDARD

NORMAL CURVE TO THE RIGHT OF THE ORDINATE

T	A	T	A	T	A	T	A
.00	.5000000	.22	.4129356	.44	.3299686	.66	.2546269
.01	.4960106	.23	.4090459	.45	.3263552	.67	.2514289
.02	.4920217	.24	.4051651	.46	.3227581	.68	.2482522
.03	.4880335	.25	.4012937	.47	.3191775	.69	.2450971
.04	.4840466	.26	.3974319	.48	.3156137	.70	.2419637
.05	.4800612	.27	.3935801	.49	.3120669	.71	.2388521
.06	.4760778	.28	.3897388	.50	.3085375	.72	.2357625
.07	.4720968	.29	.3859081	.51	.3050257	.73	.2326951
.08	.4681186	.30	.3820886	.52	.3015318	.74	.2296500
.09	.4641436	.31	.3782805	.53	.2980560	.75	.2266274
.10	.4601722	.32	.3744842	.54	.2945985	.76	.2236273
.11	.4562047	.33	.3707000	.55	.2911597	.77	.2206499
.12	.4522416	.34	.3669283	.56	.2877397	.78	.2176954
.13	.4482832	.35	.3631693	.57	.2843388	.79	.2147639
.14	.4443300	.36	.3594236	.58	.2809573	.80	.2118554
.15	.4403823	.37	.3556912	.59	.2775953	.81	.2089701
.16	.4364405	.38	.3519727	.60	.2742531	.82	.2061081
.17	.4325051	.39	.3482683	.61	.2709309	.83	.2032694
.18	.4285763	.40	.3445783	.62	.2676289	.84	.2004542
.19	.4246546	.41	.3409030	.63	.2643473	.85	.1976625
.20	.4207403	.42	.3372427	.64	.2610863	.86	.1948945
.21	.4168338	.43	.3335978	.65	.2578461	.87	.1921502

T	A	T	A	T	A	T	A
.88	.1894297	1.16	.1230244	1.44	.0749337	1.72	.0427162
.89	.1867329	1.17	.1210005	1.45	.0735293	1.73	.0418151
.90	.1840601	1.18	.1190001	1.46	.0721450	1.74	.0409295
.91	.1814113	1.19	.1170232	1.47	.0707809	1.75	.0400592
.92	.1787864	1.20	.1150697	1.48	.0694366	1.76	.0392039
.93	.1761855	1.21	.1131394	1.49	.0681121	1.77	.0383636
.94	.1736088	1.22	.1112324	1.50	.0668072	1.78	.0375380
.95	.1710561	1.23	.1093486	1.51	.0655217	1.79	.0367270
.96	.1685276	1.24	.1074877	1.52	.0642555	1.80	.0359303
.97	.1660203	1.25	.1056498	1.53	.0630084	1.81	.0351479
.98	.1635431	1.26	.1038347	1.54	.0617802	1.82	.0343795
.99	.1610871	1.27	.1020423	1.55	.0605708	1.83	.0336250
1.00	.1586553	1.28	.1002726	1.56	.0593799	1.84	.0328841
1.01	.1562476	1.29	.0985253	1.57	.0582076	1.85	.0321568
1.02	.1538642	1.30	.0968005	1.58	.0570534	1.86	.0314428
1.03	.1515050	1.31	.0950979	1.59	.0559174	1.87	.0307419
1.04	.1491700	1.32	.0934175	1.60	.0547993	1.88	.0300540
1.05	.1468591	1.33	.0917591	1.61	.0536989	1.89	.0293790
1.06	.1445723	1.34	.0901227	1.62	.0526161	1.90	.0287166
1.07	.1423097	1.35	.0885080	1.63	.0515507	1.91	.0280666
1.08	.1400711	1.36	.0869150	1.64	.0505026	1.92	.0274289
1.09	.1378566	1.37	.0853435	1.65	.0494715	1.93	.0268034
1.10	.1356661	1.38	.0837933	1.66	.0484572	1.94	.0261898
1.11	.1334995	1.39	.0822644	1.67	.0474597	1.95	.0255881
1.12	.1313569	1.40	.0807567	1.68	.0464787	1.96	.0249979
1.13	.1292381	1.41	.0792698	1.69	.0455140	1.97	.0244192
1.14	.1271432	1.42	.0778038	1.70	.0445655	1.98	.0238518
1.15	.1250719	1.43	.0763585	1.71	.0436329	1.99	.0232955

T	A	T	A	T	A	T	A
2.00	.0227501	2.26	.0119106	2.52	.0058677	2.78	.0027179
2.01	.0222156	2.27	.0116038	2.53	.0057031	2.79	.0026354
2.02	.0216917	2.28	.0113038	2.54	.0055426	2.80	.0025551
2.03	.0211783	2.29	.0110107	2.55	.0053861	2.81	.0024771
2.04	.0206752	2.30	.0107241	2.56	.0052336	2.82	.0024012
2.05	.0201822	2.31	.0104441	2.57	.0050849	2.83	.0023274
2.06	.0196993	2.32	.0101704	2.58	.0049400	2.84	.0022557
2.07	.0192262	2.33	.0099031	2.59	.0047988	2.85	.0021860
2.08	.0187628	2.34	.0096419	2.60	.0046612	2.86	.0021182
2.09	.0183089	2.35	.0093867	2.61	.0045271	2.87	.0020524
2.10	.0178644	2.36	.0091375	2.62	.0043965	2.88	.0019884
2.11	.0174292	2.37	.0088940	2.63	.0042692	2.89	.0019262
2.12	.0170030	2.38	.0086563	2.64	.0041453	2.90	.0018658
2.13	.0165858	2.39	.0084242	2.65	.0040246	2.91	.0018071
2.14	.0161774	2.40	.0081975	2.66	.0039070	2.92	.0017502
2.15	.0157776	2.41	.0079763	2.67	.0037926	2.93	.0016948
2.16	.0153863	2.42	.0077603	2.68	.0036811	2.94	.0016411
2.17	.0150034	2.43	.0075494	2.69	.0035726	2.95	.0015889
2.18	.0146287	2.44	.0073436	2.70	.0034670	2.96	.0015382
2.19	.0142621	2.45	.0071428	2.71	.0033642	2.97	.0014890
2.20	.0139034	2.46	.0069469	2.72	.0032641	2.98	.0014412
2.21	.0135526	2.47	.0067557	2.73	.0031667	2.99	.0013949
2.22	.0132094	2.48	.0065691	2.74	.0030720	3.00	.0013449
2.23	.0128737	2.49	.0063872	2.75	.0029708	3.01	.0013062
2.24	.0125455	2.50	.0062097	2.76	.0028901	3.02	.0012639
2.25	.0122245	2.51	.0060366	2.77	.0028028	3.03	.0012228

T	A	T	A	T	A	T	A
3.04	.0011829	3.28	.0005190	3.52	.0002158	3.76	.0000850
3.05	.0011442	3.29	.0005009	3.53	.0002078	3.77	.0000816
3.06	.0011067	3.30	.0004834	3.54	.0002001	3.78	.0000784
3.07	.0010703	3.31	.0004665	3.55	.0001926	3.79	.0000753
3.08	.0010350	3.32	.0004501	3.56	.0001854	3.80	.0000723
3.09	.0010008	3.33	.0004342	3.57	.0001785	3.81	.0000695
3.10	.0009676	3.34	.0004189	3.58	.0001718	3.82	.0000667
3.11	.0009354	3.35	.0004041	3.59	.0001653	3.83	.0000641
3.12	.0009043	3.36	.0003897	3.60	.0001591	3.84	.0000615
3.13	.0008740	3.37	.0003758	3.61	.0001531	3.85	.0000591
3.14	.0008447	3.38	.0003624	3.62	.0001473	3.86	.0000567
3.15	.0008164	3.39	.0003495	3.63	.0001417	3.87	.0000544
3.16	.0007888	3.40	.0003369	3.64	.0001363	3.88	.0000522
3.17	.0007622	3.41	.0003248	3.65	.0001311	3.89	.0000501
3.18	.0007364	3.42	.0003131	3.66	.0001261	3.90	.0000481
3.19	.0007114	3.43	.0003018	3.67	.0001213	3.91	.0000461
3.20	.0006871	3.44	.0002909	3.68	.0001166	3.92	.0000443
3.21	.0006637	3.45	.0002803	3.69	.0001121	3.93	.0000425
3.22	.0006410	3.46	.0002701	3.70	.0001078	3.94	.0000407
3.23	.0006190	3.47	.0002602	3.71	.0001036	3.95	.0000391
3.24	.0005976	3.48	.0002507	3.72	.0000996	3.96	.0000375
3.25	.0005770	3.49	.0002415	3.73	.0000957	3.97	.0000359
3.26	.0005571	3.50	.0002326	3.74	.0000920	3.98	.0000345
3.27	.0005377	3.51	.0002241	3.75	.0000884	3.99	.0000330

T	A	T	A	T	A	T	A
4.00	.0000317	4.27	.0000098	4.54	.0000028		
4.01	.0000304	4.28	.0000093	4.55	.0000027		
4.02	.0000291	4.29	.0000089	4.56	.0000026		
4.03	.0000279	4.30	.0000085	4.57	.0000024		
4.04	.0000267	4.31	.0000082	4.58	.0000023		
4.05	.0000256	4.32	.0000078	4.59	.0000022		
4.06	.0000245	4.33	.0000075	4.60	.0000021		
4.07	.0000235	4.34	.0000071	4.61	.0000020		
4.08	.0000225	4.35	.0000068	4.63	.0000019		
4.09	.0000216	4.36	.0000065	4.63	.0000018		
4.10	.0000207	4.37	.0000062	4.64	.0000017		
4.11	.0000198	4.38	.0000059	4.65	.0000017		
4.12	.0000189	4.39	.0000057	4.66	.0000016		
4.13	.0000181	4.40	.0000054	4.67	.0000015		
4.14	.0000174	4.41	.0000052				
4.15	.0000166	4.42	.0000049				
4.16	.0000159	4.43	.0000047				
4.17	.0000152	4.44	.0000045				
4.18	.0000146	4.45	.0000043				
4.19	.0000139	4.46	.0000041				
4.20	.0000133	4.47	.0000039				
4.21	.0000128	4.48	.0000037				
4.22	.0000122	4.49	.0000036				
4.23	.0000117	4.50	.0000034				
4.24	.0000112	4.51	.0000032				
4.25	.0000107	4.52	.0000031				
4.26	.0000102	4.53	.0000030				

APPENDIX 3H

Table 1

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectiv[es] in a Finite Population of 40 Members.

Number of Observed Defectives	Sample Size									
	2		4		8		16		32	
	90	95	90	95	90	95	90	95	90	95
0	26	30	16	20	9	11	4	5	1	1
1	37	38	26	29	15	17	7	8	2	3
2	40	40	33	35	20	22	10	11	4	4
3			38	39	25	27	13	14	5	6
4			40	40	29	31	16	17	7	7
5					33	34	18	20	8	9
6					36	37	21	22	9	10
7					39	39	23	24	11	11
8					40	40	25	27	12	13
9							28	29	13	14
10							30	31	15	15
11							32	33	16	16
12							34	35	17	18
13							36	37	18	19
14							38	38	20	20
15							39	39	21	21
16							40	40	22	23
17									23	24
18									25	25
19									26	26
20									27	27
21									28	29
22									29	30
23									31	31
24									32	32
25									33	33
26									34	34
27									35	35
28									36	36
29									37	37
30									38	38
31									39	39
32									40	40

APPENDIX 3H

Table 2

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectives in a Finite Population of 60 Members.

Number of Observed Defectives	Sample Size									
	3		6		12		24		48	
	90	95	90	95	90	95	90	95	90	95
0	31	37	18	22	9	11	4	5	1	1
1	47	51	29	33	16	18	7	9	2	3
2	57	58	39	42	21	24	10	12	4	4
3	60	60	47	50	27	30	13	15	5	6
4			54	55	32	35	16	18	7	7
5			58	59	37	39	19	21	8	9
6			60	60	41	44	22	23	10	10
7					46	48	24	26	11	12
8					50	51	27	29	12	13
9					53	54	29	31	14	14
10					56	57	32	33	15	16
11					59	59	34	36	16	17
12					60	60	37	38	17	18
13							39	41	19	19
14							41	43	20	21
15							44	45	21	22
16							46	47	23	23
17							48	49	24	25
18							50	51	25	26
19							52	53	26	27
20							54	55	28	28

Table 2 (Continued)

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectives in A Finite Population of 60 Members.

Number of Observed Defectives	Sample Size									
	3		6		12		24		48	
	90	95	90	95	90	95	90	95	90	95
21							56	57	29	30
22							58	58	30	31
23							59	59	31	32
24							60	60	33	33
25									34	34
26									35	36
27									36	37
28									37	38
29									39	39
30									40	40
31									41	42
32									42	43
33									44	44
34									45	45
35									46	46

Table 3

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectives in a Finite Population of 100 Members.

Number of Observed Defectives	Sample Size									
	5		10		20		40		80	
	90	95	90	95	90	95	90	95	90	95
0	36	44	19	24	9	12	4	5	1	1
1	57	64	32	38	16	19	7	9	3	3
2	74	80	43	49	23	26	11	12	4	5
3	88	91	54	59	28	32	14	15	5	6
4	97	98	63	68	34	38	17	19	7	7
5	100	100	72	76	39	43	19	21	8	9
6			80	84	45	48	22	24	10	10
7			87	90	50	53	25	27	11	12
8			94	95	55	58	28	30	12	13
9			98	99	60	63	30	33	14	14
10			100	100	64	68	33	35	15	16
11					69	72	36	38	16	17
12					73	76	38	41	18	18
13					78	80	41	43	19	20
14					82	84	44	46	20	21
15					86	88	46	48	22	22
16					90	91	49	51	23	24
17					93	95	51	53	24	25
18					97	97	54	56	25	26
19					99	99	56	58	27	28
20					100	100	58	61	28	29
21							61	63	29	30
22							63	65	31	32
23							66	68	32	33
24							68	70	33	34
25							70	72	34	35
26							73	75	36	37
27							75	77	37	38
28							77	79	38	39
29							80	81	40	40
30							82	83	41	42
31							84	85	42	43
32							86	87	43	44
33							88	89	45	45
34							90	91	46	47
35							92	93	47	48

Table 4

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectives in a Finite Population of 200 Members.

Number of Observed Defectives	Sample Size									
	10		20		40		80		160	
	90	95	90	95	90	95	90	95	90	95
0	40	50	20	26	10	12	4	5	1	1
1	66	77	34	41	17	20	8	9	3	3
2	88	100	47	54	23	27	11	13	4	5
3	109	120	59	66	30	34	14	16	5	6
4	128	138	70	78	36	40	17	19	7	8
5	145	154	81	89	42	46	20	22	8	9
6	161	169	91	99	47	52	23	25	10	10
7	176	181	102	109	53	58	26	28	11	12
8	188	192	112	119	58	63	29	31	12	13
9	197	198	121	128	64	69	31	34	14	15
10	200	200	131	137	69	74	34	37	15	16
11			140	146	74	80	37	40	16	17
12			149	155	80	85	40	42	18	19
13			157	163	85	90	42	45	19	20
14			165	170	90	95	45	48	20	21
15			173	178	95	100	48	51	22	23
16			181	184	100	105	50	53	23	24
17			188	190	105	110	53	56	22	23
18			194	196	110	115	56	59	26	27
19			198	199	115	120	58	61	27	28
20			200	200	120	125	61	64	28	29
21					125	129	64	67	30	31
22					129	134	66	69	31	32
23					134	139	69	72	32	33
24					139	143	71	74	34	35
25					143	148	74	77	35	36
26					148	152	77	80	36	37
27					153	157	79	82	37	39
28					157	161	82	85	39	40
29					161	165	84	87	40	41
30					165	169	87	90	41	43
31					170	173	89	92	43	44
32					174	177	92	95	44	45
33					178	181	94	97	45	46
34					182	185	97	100	47	48
35					186	188	99	102	48	49

Table 5

Upper 90- and 95- Percent Confidence Bounds for the Number of Defects in a Finite Population of 240 Members.

Number of Observed Defectives	Sample Size									
	12		24		48		96		192	
	90	95	90	95	90	95	90	95	90	95
0	40	51	20	28	10	13	4	5	1	1
1	67	79	35	42	17	21	8	9	3	3
2	91	103	48	55	24	28	11	13	4	5
3	112	125	60	68	30	34	14	16	5	6
4	133	144	71	80	36	41	17	19	7	8
5	152	163	83	91	42	47	20	22	8	9
6	169	179	93	102	48	53	23	25	10	10
7	186	195	104	112	53	58	26	28	11	12
8	202	209	114	123	59	64	29	31	12	13
9	216	222	124	133	64	70	31	34	14	15
10	223	232	134	142	70	75	34	37	15	16
11	237	239	144	152	75	81	37	40	16	17
12	240	240	153	161	81	86	40	43	18	19
13			163	170	86	92	43	45	19	20
14			172	179	91	97	45	48	20	21
15			180	187	96	102	48	51	22	23
16			189	195	102	107	51	54	23	23
17			198	203	107	112	53	56	24	25
18			206	211	112	117	56	59	26	27
19			213	218	117	122	59	62	27	28
20			221	224	122	127	61	65	28	29
21			228	230	127	132	64	67	30	31
22			234	236	132	137	67	70	31	32
23			239	239	137	142	69	73	32	33
24			240	240	142	147	72	75	34	35
25					146	152	74	78	35	36
26					151	156	77	80	36	37
27					156	161	80	83	38	39
28					161	166	82	86	39	40
29					165	170	85	88	40	41
30					170	175	87	91	41	43
31					175	179	90	93	43	44
32					179	184	93	96	44	45
33					184	188	95	99	45	47
34					188	193	98	101	47	48
35					193	197	100	104	48	49

APPENDIX 3H

Table 6

Upper 90- and 95- Percent Confidence Bounds for the Number of Defectives in a Finite Population of 300 Members.

Number of Observed Defectives	Sample Size									
	15		30		60		120		240	
	90	95	90	95	90	95	90	95	90	95
0	41	53	21	27	10	13	4	5	1	1
1	69	82	35	42	17	21	8	9	3	3
2	93	107	48	56	24	28	11	13	4	5
3	116	130	61	69	30	34	14	16	5	6
4	137	151	73	81	36	41	17	19	7	8
5	158	171	84	93	42	47	20	22	8	9
6	177	190	95	104	48	53	23	25	10	10
7	196	208	106	115	54	59	26	28	11	12
8	214	225	117	126	60	65	29	31	12	13
9	231	241	127	137	65	71	32	34	14	15
10	247	256	138	147	71	76	34	37	15	16
11	262	270	148	157	76	82	37	40	16	17
12	276	282	158	167	82	88	40	43	18	19
13	288	292	168	177	87	93	43	46	19	20
14	297	299	177	186	92	98	45	49	21	21
15	300	300	187	196	98	104	48	51	22	23
16			196	205	103	109	51	54	23	24
17			206	214	108	114	54	57	24	26
18			215	223	113	120	56	60	26	27
19			224	231	119	125	59	62	27	28
20			233	240	124	130	62	65	28	30
21			241	248	129	135	64	68	30	31
22			250	256	134	140	67	70	31	32
23			285	264	139	145	70	73	32	34
24			266	271	144	150	72	76	34	35
25			274	278	149	155	75	79	35	36
26			281	285	154	160	78	81	36	37
27			288	291	159	165	80	84	38	39
28			294	296	164	170	83	87	39	40
29			298	299	169	175	86	89	40	41
30			300	300	174	180	88	92	42	43
31					179	185	91	94	43	44
32					184	190	93	97	44	45
33					189	194	96	100	45	47
34					193	199	99	102	47	48
35					198	204	101	105	48	49

Table 7

Upper 90- and 95- Percent Confidence Bounds for the Number of Defectives in a Finite Population of 360 Members.

Number of Observed Defectives	Sample Size									
	18		36		72		144		288	
	90	95	90	95	90	95	90	95	90	95
0	42	53	21	27	10	13	4	5	1	1
1	70	84	36	43	17	21	8	9	3	3
2	95	110	49	57	24	28	11	13	4	5
3	119	133	61	70	30	35	14	16	5	6
4	141	156	73	82	36	41	17	19	7	8
5	162	177	85	94	42	47	20	22	8	9
6	182	197	96	106	48	54	23	25	10	10
7	202	217	108	117	54	60	26	28	11	12
8	221	235	118	129	60	66	29	31	12	13
9	240	253	129	139	66	71	32	34	14	15
10	258	270	140	150	71	77	35	37	15	16
11	275	287	150	161	77	83	37	40	17	17
12	292	302	161	171	82	88	40	43	18	19
13	308	317	171	181	88	94	43	46	19	20
14	323	330	181	191	93	100	46	49	21	22
15	336	342	191	201	99	105	48	52	22	23
16	348	352	201	211	104	110	51	54	23	24
17	357	359	211	220	109	116	54	57	25	26
18	360	360	220	230	115	121	57	60	26	27
19			230	239	120	127	59	63	27	28
20			239	248	125	132	62	65	28	30
21			248	257	130	137	65	68	30	31
22			258	266	135	142	67	71	31	32
23			267	275	141	148	70	74	32	34
24			276	284	146	153	73	76	34	35
25			285	292	151	158	75	79	35	36
26			293	300	156	163	78	82	36	38
27			302	308	161	168	81	84	38	39
28			310	316	166	173	83	87	39	40
29			318	324	171	178	86	90	40	42
30			326	331	176	183	89	92	42	43
31			334	338	181	188	91	95	43	44
32			341	345	186	193	94	98	44	45
33			348	351	191	198	96	100	45	47
34			354	356	196	203	99	103	47	48
35			359	359	201	208	102	106	48	49

Table 8

Upper 90- and 95-Percent Confidence Bounds for the Number of Defectives in a Finite Population of 400 Members.

Number of Observed Defectives	Sample Size									
	20		40		80		160		320	
	90	95	90	95	90	95	90	95	90	95
0	42	54	21	27	10	13	4	5	1	1
1	71	84	36	43	17	21	8	9	3	3
2	96	111	49	57	24	28	11	13	4	5
3	120	135	62	70	30	35	14	16	5	6
4	142	158	74	83	37	41	17	19	7	8
5	164	180	86	95	43	48	20	22	8	9
6	185	201	97	107	49	54	23	26	10	10
7	205	221	108	119	54	60	26	29	11	12
8	225	240	119	130	60	66	29	32	12	13
9	244	259	130	141	66	72	32	34	14	15
10	263	277	141	152	71	77	35	37	15	16
11	281	294	152	162	77	83	37	40	17	17
12	299	311	162	173	83	89	40	43	18	19
13	316	327	172	183	88	94	43	46	19	20
14	332	342	183	194	94	100	46	49	21	22
15	348	357	193	204	99	106	48	52	22	23
16	363	370	203	214	104	111	51	54	23	24
17	376	382	213	224	110	117	54	57	25	26
18	388	392	223	233	115	122	57	60	26	27
19	397	399	233	243	120	127	59	63	27	28
20	400	400	242	253	126	133	62	66	29	30
21			252	262	131	138	65	68	30	31
22			261	271	136	143	68	71	31	32
23			271	280	141	149	70	74	32	34
24			280	289	147	154	73	77	34	35
25			289	298	152	159	76	79	35	36
26			298	307	157	164	78	82	36	38
27			307	316	162	169	81	85	38	39
28			316	324	167	174	84	87	39	40
29			325	333	172	180	86	90	40	42
30			333	341	177	185	89	93	42	43
31			342	349	183	190	91	95	43	44
32			350	357	188	195	94	98	44	46
33			358	364	193	200	97	101	46	47
34			366	371	198	205	99	103	47	48
35			374	378	203	210	102	106	48	49

Table 9

Upper 90- and 95-Percent Confidence Bounds for the Number of

Defectives in a Finite Population of 500 Members.

Number of Observed Defectives	Sample Size									
	25		50		100		200		400	
	90	95	90	95	90	95	90	95	90	95
0	42	55	21	27	10	13	4	5	1	1
1	72	86	36	43	17	21	8	9	3	3
2	98	113	49	58	24	28	11	13	4	5
3	122	138	62	71	30	35	14	16	5	6
4	145	162	74	84	37	42	17	19	7	8
5	168	185	86	96	43	48	20	22	8	9
6	190	207	98	109	49	54	23	26	10	10
7	211	229	110	120	55	60	26	29	11	12
8	232	249	121	132	60	66	29	32	12	13
9	252	269	132	143	66	72	32	35	14	15
10	272	289	143	155	72	78	35	38	15	16
11	291	308	154	166	78	84	37	40	17	17
12	310	327	165	176	83	90	40	43	18	19
13	329	345	175	187	89	95	43	46	19	20
14	348	363	186	198	94	101	46	49	21	22
15	366	380	196	208	100	107	49	52	22	23
16	383	397	207	219	105	112	51	55	23	24
17	400	413	217	229	111	118	54	58	25	26
18	417	428	227	239	116	123	57	60	26	27
19	433	443	237	250	121	129	60	63	27	28
20	448	457	247	260	127	134	62	66	29	30
21	463	470	257	270	132	140	65	69	30	31
22	477	482	267	279	137	145	68	71	31	32
23	489	492	277	289	143	150	70	74	32	34
24	497	499	287	299	148	156	73	77	34	35
25	500	500	297	309	153	161	76	80	35	36
26			307	318	159	166	79	82	36	38
27			316	327	164	172	81	85	38	39
28			326	337	169	177	84	88	39	40
29			335	346	174	182	87	91	40	42
30			345	355	179	187	89	93	42	43
31			354	364	185	193	92	96	43	44
32			363	373	190	198	94	99	44	46
33			372	382	195	203	97	101	46	47
34			381	391	200	208	100	104	47	48
35			390	400	205	213	102	107	48	50

FACTORIAL TREATMENT PROCEDURE WORKSHEETS

Table 1

4 TREATMENTS AND 8 ITEMS

Design: $1/2 \times 2^4$ (Ref. 15, page 484)

Treatments	Item numbers							
	1	2	3	4	5	6	7	8
A	NONE	+	+		+			+
B		+		+		+		+
C			+	+'			+	+
D					+	+	+	+
Results								

Remarks:

1. All main effects are clear of two-factor interactions.

2. Two-factor interactions are confused with one another and are not measurable.

3. Three-factor and higher order interactions are assumed negligible.

Table 2

5 TREATMENTS AND 8 ITEMS

Design: $1/4 \times 2^5$ (Ref. 15, page 484)

Treatments	Item numbers							
	1	2	3	4	5	6	7	8
A	NONE	+		+	+		+	
B			+	+	+	+		
C			+	+			+	+
D		+		+		+		+
E					+	+	+	+

Results

Remarks:

1. All main effects are confused with two-factor inter-actions.

2. All interactions are assumed negligible.

APPENDIX 4

Table 3

6 TREATMENTS AND 8 ITEMS

Design: $1/8 \times 2^6$ (Ref. 15, page 485)

Treatments	Item numbers							
	1	2	3	4	5	6	7	8
A	NONE	+		+	+		+	
B			+	+			+	+
C		+		+		+		+
D			+	+	+	+		
E		+	+			+	+	
F					+	+	+	+
Results								
Remarks:								

1. All main effects are confused with two-factor interactions.

2. All interactions are assumed negligible.

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Table 4

7 TREATMENTS AND 8 ITEMS

Design: $1/16 \times 2^7$ (Ref. 15, page 485)

Treatments	Item numbers							
	1	2	3	4	5	6	7	8
A		+	+	+	+			
B		+	+			+	+	
C		+		+		+		+
D	NONE	+			+		+	+
E			+	+			+	+
F			+		+	+		+
G				+	+	+	+	
Results								
Remarks:								

1. All main effects are confused with two-factor inter-actions.

2. All interactions are assumed negligible.

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APPENDIX 4

Table 5

7 TREATMENTS AND 8 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers								
	1	2	3	4	5	6	7	8
A	NONE			+		+	+	+
B			+		+	+	+	
C		+		+	+	+		
D			+	+	+			+
E		+	+	+			+	
F		+	+			+		+
G		+			+		+	+
Results								

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

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APPENDIX 4

Table 6

8 TREATMENTS AND 12 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	1	2	3	4	5	6	7	8	9	10	11	12
						Item numbers						
A			+				+	+	+		+	+
B		+				+	+	+		+	+	
C					+	+	+		+	+		+
D	NONE			+	+	+		+	+		+	
E			+	+	+		+	+		+		
F		+	+	+		+	+		+			
G		+	+			+	+	+				+
H		+		+	+		+				+	+

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negigible.

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Table 7

9 TREATMENTS AND 12 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers											
	1	2	3	4	5	6	7	8	9	10	11	12
A	NONE		+				+	+	+		+	+
B		+				+	+	+		+	+	
C				+		+	+		+	+		+
D				+	+	+		+	+		+	
E			+	+	+		+	+		+		
F		+	+	+		+	+		+			
G		+	+		+	+		+				+
H		+		+	+		+				+	+
I			+	+		+				+	+	+
Results												

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

Table 8

10 TREATMENTS AND 12 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers.											
	1	2	3	4	5	6	7	8	9	10	11	12
A	NONE		+				+	+	+		+	+
B	NONE	+				+	+	+		+	+	
C	NONE				+	+	+		+	+		+
D	NONE			+	+	+		+	+		+	
E	NONE		+	+	+		+	+		+		
F	NONE	+	+	+		+	+		+			
G	NONE	+	+		+	+		+				+
H	NONE	+		+	+		+				+	+
I	NONE		+	+		+				+	+	+
J	NONE	+	+		+				+	+	+	

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

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APPENDIX 4

Table 9

11 TREATMENTS AND 12 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments		1	2	3	4	5	6	7	8	9	10	11	12
						Item numbers							
A				+				+	+	+		+	+
B			+				+	+	+		+	+	
C						+	+	+		+	+		+
D					+	+	+		+	+		+	
E	NONE			+	+	+		+	+		+		
F			+	+	+		+	+		+			
G			+	+		+	+		+				+
H			+		+	+		+				+	+
I				+	+		+				+	+	+
J			+	+		+				+	+	+	
K			+		+				+	+	+		+
Results													
Remarks:													

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

Table 10

5 TREATMENTS AND 16 ITEMS

Design: $1/2 \times 2^5$ (Ref. 6, page 5)

Treatments	Item numbers															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE	+		+	+		+		+		+			+		+
B		+		+	+		+			+		+	+		+	
C		+		+		+		+	+		+		+		+	
D		+	+			+	+			+	+			+	+	
E			+	+			+		+		+	+			+	+
Blocks	I								II							
Results																

Remarks:

1. All main effects are clear of two-factor interactions.

2. When blocks are not used all two-factor interactions are measurable. When blocks are used the AB interaction is not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 11

5 TREATMENTS AND 16 ITEMS

Design: $1/2 \times 2^5$ (Ref. 6, page 5)

Treatments	Item numbers															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE	+	+		+			+	+			+		+	+	
B		+	+		+			+		+	+		+			+
C		+	+			+	+		+			+	+			+
D		+		+	+		+		+		+			+		+
E			+	+	+	+			+	+					+	+
Blocks	I				II				III				IV			
Results																

Remarks:

1. All main effects are clear of two-factor interactions.

2. When blocks are not used, all two-factor interactions are measurable.

   When blocks are used, interactions AB, AC, and BC are not measurable.

3. All three-factor and higher order interactions are considered negligible.

Table 12

6 TREATMENTS AND 16 ITEMS

Design: 1/4 X 2^6 (Factorial (Ref. 6, page 18)

Treatments		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
							Item numbers										
A			+	+		+			+	+			+		+	+	
B			+	+			+	+		+			+	+			+
C	NONE		+		+	+		+		+		+			+		+
D			+		+		+		+	+		+		+		+	
E				+	+			+	+	+	+	+		+	+		
F				+	+	+	+			+	+					+	+
Blocks		I								II							

Results

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assume negligible.

Table 13

6 TREATMENTS AND 16 ITEMS

Design: $1/4 \times 2^6$ Factorial (Ref. 6, page 18)

Treatments	Item numbers															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE	+	+		+			+	+			+		+	+	
B		+	+		+			+		+	+		+			+
C		+		+	+		+		+		+			+		+
D		+		+	+		+			+		+	+		+	
E			+	+	+	+					+	+	+	+		
F			+	+	+	+			+	+					+	+
Blocks	I				II				III				IV			
Results																

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 14

6 TREATMENTS AND 16 ITEMS

Design: 1/4 X 2^6 Factorial (Ref. 6, page 18)

Treatments		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
							Item numbers										
A	NONE		+		+	+		+			+		+	+		+	
B	NONE		+		+	+		+		+		+			+		+
C	NONE		+	+			+	+				+			+	+	
D	NONE		+	+		+			+			+		+			+
E	NONE		+	+			+	+		+			+	+			+
F	NONE		+	+		+			+	+			+		+	+	

Blocks	I		II	III	IV		V		VI		VII		VIII	

| Results | | | | | | | | | | | | | | | | |

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 15

7 TREATMENTS AND 16 ITEMS

Design: 1/8 X 2^7 Factorial (Ref. 6, page 30)

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE	+	+		+			+		+	+		+			+
B		+	+		+			+	+			+		+	+	
C		+	+			+	+			+	+			+	+	
D		+	+			+	+		+			+	+			+
E		+		+	+		+			+		+	+		+	
F		+		+		+		+	+		+		+		+	
G		+		+	+		+		+		+			+		+

Blocks	I	II

Results																	

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 16

7 TREATMENTS AND 16 ITEMS

Design: $1/8 \times 2^7$ Factorial (Ref. 6, page 30)

Treatments	Item numbers															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE	+	+		+			+		+	+		+			+
B		+	+		+			+	+			+		+	+	
C		+	+			+	+			+	+			+	+	
D		+	+			+	+		+			+	+			+
E		+		+	+		+			+		+	+		+	
F		+		+		+		+	+		+		+		+	
G		+		+	+		+		+		+			+		+
Blocks	I				II				III				IV			
Results																

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 17

8 TREATMENTS AND 16 ITEMS

Design: $1/16 \times 2^8$ Factorial (Ref. 6, page 42)

Treatments		Item numbers															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A				+	+	+	+			+	+					+	+
B				+	+	+	+					+	+	+	+		
C	NONE		+	+		+			+	+			+		+	+	
D			+	+		+			+		+	+		+			+
E			+		+	+		+			+		+	+		+	
F			+		+	+		+		+		+			+		+
G						+	+	+	+	+	+	+	+				
H						+	+	+	+					+	+	+	+

Blocks		I								II							
Results																	

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

APPENDIX 4

Table 18

8 TREATMENTS AND 16 ITEMS

Design: 1/16 X 2^8 Factorial (Ref. 6, page 41)

Treatments	Item numbers															
---	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A		+	+		+			+		+	+		+			+
B		+	+			+	+			+	+			+	+	
C		+	+		+			+	+			+		+	+	
D		+	+			+	+		+			+	+			+
E	NONE		+	+			+	+	+	+			+	+		
F			+	+	+	+			+	+					+	+
G			+	+	+	+					+	+	+	+		
H			+	+			+	+			+	+			+	+
Blocks	I				II				III				IV		
Results																

Remarks:

1. All main effects are clear of two-factor interactions.

2. All two-factor interactions are confused with one another and are not measurable.

3. All three-factor and higher order interactions are assumed negligible.

Table 19

9 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	NONE				+			+	+		+		+	+	+	+
B			+				+	+		+		+	+	+	+	
C			+			+	+		+		+	+	+	+		
D		+			+	+		+		+	+	+	+			
E				+	+			+	+	+	+	+				+
F			+	+		+		+	+	+	+				+	
G		+	+		+		+	+	+	+				+		
H		+		+		+	+	+	+				+			+
I			+		+	+	+	+				+			+	+

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

APPENDIX 4

Table 20

10 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
															Item numbers (cols 1–16)	
A	NONE				+			+	+		+		+	+	+	+
B				+			+	+		+		+	+	+	+	
C			+			+	+		+		+	+	+	+		
D		+			+	+		+		+	+	+	+			
E				+	+		+		+	+	+	+				+
F			+	+		+		+	+	+	+				+	
G		+	+		+		+	+	+	+				+		
H		+			+		+	+	+	+			+			+
I			+		+	+	+	+				+			+	+
J		+		+	+	+	+				+			+	+	

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

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Table 21

11 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
													Item numbers (header span)			
A	NONE				+			+	+		+		+	+	+	+
B			+				+	+		+		+	+	+	+	
C			+			+	+		+		+	+	+	+		
D		+			+	+		+		+	+	+	+			
E				+	+		+		+	+	+	+				+
F			+	+		+		+	+	+	+				+	
G		+	+			+		+	+	+	+			+		
H		+		+		+	+	+	+				+			+
I			+		+	+	+	+	+			+			+	+
J		+		+	+	+	+	+			+			+	+	
K			+	+	+	+				+			+	+		+
Results																

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

Table 22

12 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers															
	1	2	3	4	5	6	7	8	9	1C	11	12	13	14	15	16
A					+			+	+		+		+	+	+	+
B				+			+	+			+	+	+	+	+	
C			+			+	+		+		+	+	+	+		
D		+			+	+		+			+	+	+	+		
E				+	+		+		+		+	+	+			+
F			+	+		+		+	+		+	+			+	
G	NONE	+	+		+		+	+	+		+			+		
H		+		+		+	+	+	+				+			+
I			+		+	+	+	+				+			+	+
J		+		+	+	+	+				+			+	+	
K			+	+	+	+				+			+	+		+
L		+	+	+	+				+			+	+		+	

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

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Table 23

13 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments									Item numbers							
	1	2	3	4	5	6	7	8	9	1C	11	12	13	14	15	16
A					+			+	+		+		+	+	+	+
B			+				+	+		+		+	+	+	+	
C			+			+		+	+		+		+	+	+	
D		+			+	+		+		+	+	+	+			
E				+	+		+		+	+	+	+				+
F			+	+		+		+	+	+	+			+		
G		+	+			+		+	+	+	+				+	
H	NONE	+		+		+	+	+	+				+			+
I			+		+	+	+	+				+			+	+
J		+		+	+	+	+				+			+	+	
K			+	+	+	+				+			+	+		+
L		+	+	+	+				+			+	+		+	
M		+	+	+				+			+	+		+		+
Results																

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

Table 24

14 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments																

Treatments	1 (NONE)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A					+			+	+		+		+	+	+	+
B				+			+	+		+		+	+	+	+	
C			+			+	+		+		+	+	+	+		
D		+			+	+		+		+	+	+	+			
E				+	+		+		+	+	+	+				+
F			+	+		+		+	+	+	+				+	
G		+	+		+		+	+	+	+				+		
H		+		+		+	+	+	+				+			+
I			+		+	+	+	+				+			+	+
J		+		+	+	+	+				+			+	+	
K			+	+	+	+				+			+	+		+
L		+	+	+					+			+	+		+	
M		+	+	+				+			+	+		+		+
N		+	+				+			+	+		+		+	+

Results

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

Table 25

15 TREATMENTS AND 16 ITEMS

Design: Multifactorial (Ref. 5, page 323)

Treatments		Item numbers															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A						+			+	+		+		+	+	+	+
B					+			+	+		+		+	+	+	+	
C				+			+	+			+	+	+	+	+		
D			+			+	+		+		+	+	+	+			
E					+	+		+		+	+	+	+				+
F				+	+		+		+	+	+	+				+	
G			+	+		+		+	+	+	+				+		
H	NONE		+		+		+	+	+	+				+			+
I				+		+	+	+	+				+			+	+
J			+		+	+	+	+			+				+	+	
K				+	+	+	+				+			+	+		+
L			+	+	+	+				+			+	+		+	
M			+	+	+				+			+	+		+		+
N			+	+				+			+	+		+		+	+
O			+				+			+	+		+		+	+	+
Results																	

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

3. The treatment combinations in the individual rows of this design can be used in any combination of two or more, up to and including 15 treatments.

APPENDIX 4

Table 26

19 Treatments and 20 Items

Design: Multifactorial (Ref. 5, page 323)

Treatments	Item numbers																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A			+	+					+		+		+	+	+	+			+	+
B		+	+					+		+		+	+	+	+			+	+	
C		+				+			+		+	+	+	+			+	+		+
D					+			+		+	+	+	+			+	+		+	+
E				+			+		+	+	+	+			+		+		+	+
F			+			+		+	+	+	+	+		+	+		+	+		
G		+		+			+	+	+	+				+	+		+	+		
H		+		+		+	+	+	+			+	+			+	+			
I		+		+		+	+	+				+	+		+	+				+
J		+		+	+	+	+			+	+			+	+				+	
K			+	+	+	+			+	+			+	+				+		+
L		+	+	+	+				+	+		+	+				+		+	
M		+	+	+			+	+			+	+				+		+		+
N		+	+				+	+		+	+					+		+	+	+
O		+			+	+			+	+					+		+	+	+	+
P			+	+			+	+					+		+		+	+	+	+
Q			+	+			+	+			−	+		+		+	+	+	+	
R		+	+		+	+					+			+		+	+	+	+	
S		+		+	+					+		+		+	+	+	+			+
Results																				

Remarks:

1. All main effects are confused with interactions.

2. All interactions are assumed negligible.

3. The treatment combinations in the individual rows of this design can be used in any combination of two or more, up to and including 19 treatments.

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Table 27
23 Treatment and 24 Items
Design: Multifactorial (Ref. 5, page 323)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A						+	+	+			+	+			+	+		+		+	+	+	+	+
B		+		+	+		+			+	+			+	+		+			+	+	+	+	
C				+		+			+	+			+			+		+	+	+	+	+		
D			+		+				+			+	+				+	+	+	+	+			
E		+	+	+			+	+			+	+	+	+	+	+	+	+	+	+				+
F			+		+		+	+	+	+	+	+	+		+	+	+	+	+	+			+	+
G		+				+			+	+		+	+	+	+	+	+	+				+	+	+
H				+			+			+	+		+	+	+	+	+				+			+
I		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+
J		+	+		+		+	+	+	+	+	+	+	+										
K		+	+	+	+	+		+	+	+	+	+	+	+			+	+	+				+	+
L		+	+	+		+	+	+	+	+	+	+	+				+	+	+	+		+	+	+
M				+	+	+	+	+	+	+	+	+			+		+							
N		+	+	+	+	+	+	+			+	+	+	+		+	+	+	+	+	+		+	+
O		+	+	+	+	+	+	+	+				+	+	+					+	+	+	+	+
P			+	+	+	+	+	+	+	+				+		+		+					+	+
Q				+	+	+	+	+				+	+		+			+	+		+	+	+	+
R		+	+	+	+	+	+	+	+	+	+						+	+	+		+	+	+	+
S		+	+	+	+	+				+	+		+	+	+	+	+	+	+	+	+	+	+	+
T		+	+																					
U		+	+	+			+	+	+	+	+		+	+			+	+	+	+	+	+	+	+
V		+	+	+		+	+		+			+	+	+			+		+		+	+	+	+
W		+	+																					
Results																								

Remarks:
1. All main effects are confused with interactions.
2. All interactions are assumed negligible.
3. The treatment combinations in the individual rows of this design can be used in any combination of two or more up to and including 23 treatments.

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APPENDIX 4

Table 27
23 Treatment and 24 Items
Design: Multifactorial (Ref. 5, page 323)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A		+		+	+	+		+			+	+			+	+	+	+		+	+	+	+	+
B				+			+			+	+			+	+		+		+	+	+	+	+	
C			+		+				+	+			+	+		+		+	+	+	+	+		
D		+	+	+				+	+			+	+		+	+	+	+	+	+	+			
E							+	+			+		+	+		+	+	+	+	+				
F		+		+	+	+	+		+	+	+	+	+	+	+	+	+	+	+				+	+
G			+	+		+			+	+				+	+	+	+	+				+		+
H							+	+	+		+	+	+	+	+					+			+	+
I		+	+				+	+	+	+	+	+	+	+	+	+	+	+	+	+				
J		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+							+
K		+	+	+	+	+		+	+	+	+	+	+			+				+	+	+	+	+
L		+	+	+	+	+	+	+	+	+	+	+	+							+		+	+	+
M		+	+		+	+		+	+	+	+						+	+						+
N		+						+		+	+							+						+
O			+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+				+
P		+		+	+	+						+	+	+	+	+		+	+	+	+		+	+
Q		+	+	+	+	+		+		+		+	+	+	+		+	+	+		+	+	+	+
R		+	+	+	+	+	+				+		+									+		
S										+	+	+	+		+		+		+		+	+		
T		+		+		+		+	+	+	+		+			+		+	+	+			+	+
U		+	+	+	+	+		+		+	+			+	+		+	+	+	+		+	+	+
V		+	+	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+			+	+	+
W		+	+	+	+		+		+	+		+		+		+	+	+	+	+		+	+	+
Results																								

Remarks:
1. All main effects are confused with interactions.
2. All interactions are assumed negligible.
3. The treatment combinations in the individual rows of this design can be used in any combination of two or more up to and including 23 treatments.

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APPENDIX 4

Table 29
6 Treatments and 32 Items
Design: 1/2 X 2^6 Factorial (Ref. 6, page 7)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+		+	+		+		+		+			+		+	+		+			+		+		+		+	+		+	
B		+	+		+			+	+			+		+	+			+	+		+	+		+	+			+		+	+	
C		+	+		+			+			+		+				+			+		+	+		+			+		+	+	
D		+		+		+		+		+		+		+		+		+		+		+		+		+		+		+		+
E			+	+	+	+		+			+	+	+	+		+			+	+	+	+					+	+		+		
F			+	+			+	+			+	+			+	+			+	+			+	+			+	+			+	+
Blocks								I												II												
Results																																

(Row D marked "NONE")

Remarks:
1. All main effects are clear of two-factor interactions.
2. All two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

APPENDIX 4

Table 30
6 Treatments and 32 Items
Design: 1/2 X 2⁶ Factorial (Ref. 6, page 6)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A			+	+			+	+	+	+			+	+			+	+			+	+					+	+			+	+
B		+	+			+	+		+	+	+			+	+		+			+	+			+	+			+	+			+
C		+	+			+	+		+			+	+			+		+	+			+	+		+			+	+			+
D			+	+	+	+					+	+		+		+		+				+	+	+	+	+					+	+
E		+	+	+	+	+	+		+		+		+	+		+	+	+		+	+				+	+	+			+		+
F		+		+		+		+	+		+		+		+		+		+		+		+			+				+		+
BLOCKS			I									II							III								IV					
Results																																

(NONE — appears in the D treatment column area)

Remarks:

1. All main effects are clear of two-factor interactions.
2. When blocks are not used, all two-factor interactions are measurable. When blocks are used, interaction BC is not measurable.
3. All three-factor and higher order interactions are assumed negligible.

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Table 31
6 Treatments and 32 Items
Design: 1/2 x 2^6 Factorial (Ref. 6, page 6)

Treat-ments	ITEM NUMBERS																															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A			+	+	+	+			+	+					+	+			+	+	+	+			+	+					+	+
B		+	+			+	+		+				+			+	+			+	+			+		+	+			+	+	
C		+	+					+		+	+		+			+	+		+			+	+					+		+	+	
D			+	+				+			+	+				+	+	+			+	+			+	+			+	+		
E		+	+	+	+		+		+		+			+	+	+	+	+		+	+	+		+	+	+		+	+	+	+	
F		+	+	+	+		+		+		+			+		+	+	+		+	+	+	+		+		+	+		+	+	+
BLOCKS	I				II				III				IV				V				VI				VII				VIII			
Results																																

(In the D treatment row at item 1: **NONE**)

Remarks:

1. All main effects are clear of two-factor interactions.
2. When blocks are not used, all two factor interactions are measurable.
   When blocks are used, interactions AD, BC, & EF are not measurable.
3. All three-factor and higher order interactions are assumed negligible.

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APPENDIX 4

Table 32
7 Treatments and 32 Items
Design: $1/4 \times 2^7$ Factorial (Ref. 6, page 20)

Treatments	ITEM NUMBERS																															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A					+	+	+	+	+	+	+	+								+	+	+	+	+	+	+	+	+				
B					+	+	+	+	+	+	+	+					+	+	+										+	+	+	+
C		+	+			+	+			+	+			+	+			+		+				+	+			+	+			+
D					+	+	+	+					+	+	+	+	+	+	+	+					+	+	+	+				
E		+		+		+		+		+		+		+		+	+		+			+		+		+		+	+	+		+
F					+	+	+	+					+	+	+	+					+	+	+	+					+	+	+	+
G			+	+			+	+			+	+			+	+			+	+			+	+			+	+			+	+
Blocks	I (NONE)																II															
Results																																

Remarks:

1. All main effects are free of two-factor interactions.
2. When blocks are not used interactions AB, AC, AE, BC, BE and CE are not measurable.
   When blocks are used interactions AB, AD, AF, BD, HF and DF are not measurable.
3. All three-factor and higher order interactions are assumed negligible.

7 Treatments and 32 Items
Design: 1/4 x 2^7 Factorial (Ref. 6, page 19)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+		+			+	+			+		+	+		+			+		+	+			+	+		+			+
B		+		+		+		+	+		+		+		+		+		+		+		+			+		+		+		+
C		+		+	+		+		+		+			+		+	+		+			+		+		+		+	+		+	
D			+	+			+		+	+			+	+			+	+			+	+		+			+	+			+	+
E		+	+			+	+		+			+	+			+	+			+	+			+		+	+			+	+	
F			+	+			+	+			+	+			+	+		+			+	+			+	+			+	+		
G			+	+		+			+	+					+	+	+		+	+	+	+			+	+					+	+
Blocks	NONE				I								II							III								IV				
Results																																

Remarks:
1. All main effects are free of two-factor interactions.
2. When blocks are not used interactions AB, AC, AE, BC, BE and CE are not measurable.
   When blocks are used interactions AB, AC, AE, BC, BE, CE and DF are not measurable.
3. All three-factor and higher order interactions are assumed negligible.

(Treatments and 32 Items
Design: 1/4 X 2⁷ Factorial (Ref. 6, page 19)

ITEM NUMBERS

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+			+	+		+			+	+			+	+			+	+			+		+	+			+	+	
B		+		+	+	+	+		+		+			+		+	+		+		+	+		+		+		+			+	
C		+		+	+	+		+	+		+		+		+	+	+		+		+		+			+		+		+		+
D			+	+	+	+			+	+					+		+	+	+				+	+			+	+	+	+		
E		+	+	+	+			+	+			+		+	+	+	+			+		+	+			+	+		+			+
F			+	+	+	+			+	+										+	+	+			+	+					+	+
G			+	+	+	+			+		+	+	+	+		+	+	+	+	+	+		+	+	+	+					+	+
Blocks		I				II				III				IV				V				VI				VII				VIII		
Results																																

(NONE — written vertically across center)

Remarks: 1. All main effects are clear of two-factor interactions.
2. When blocks are not used, interactions AB, AC, AE, BC, BE and CE are not measurable. When blocks are used only interactions AD, AF, AG, BD, BF, BG, CD, CF, CG, EF, EG and DE are measurable.
3. All three-factor and higher order interactions are assumed negligible.

ITEM NUMBERS

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+		+	+		+		+		+			+	+			+		+	+		+		+		+			+		+
Γ		+		+	+		+			+		+	+			+		+	+	+	+	+	+			+		+	+		+	
C		+		+	+		+		+		+			+	+		+		+			+		+	+	+		+	+		+	
D		+		+	+		+			+							+		+			+		+	+		+			+		+
E		+	+			+	+		+			+	+		+	+		+	+			+	+		+			+	+	+		+
F		+		+		+	+	+	+		+	+	+			+	+		+		+	+	+			+		+	+	+		+
G		+		+		+		+	+		+		+			+	+	+		+		+		+	+		+		+		+	
H			+	+			+	+		+				+					+	+			+	+	+	+	+		+	+		
Blocks								I															II									
Results																																

Item 1 row (Treatments): N O N E

Remarks:
1. All main effects are clear of two-factor interactions.
2. Only the following interactions are measurable: AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, FH, FH and GH whether or not blocks are used.
3. All three-factor and higher order interactions are assumed negligible.

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Table 36
8 Treatments and 32 Items
Design: 1/8 x 2^8 Factorial (Ref. 6, page 31)

ITEM NUMBERS

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A			+	+	+	+			+	+					+	+	+	+					+	+			+	+	+	+	+	
B					+	+	+	+	+	+	+	+					+	+	+	+									+	+	+	+
C		+						+	+		+	+		+			+			+		+	+			+	+		+		+	+
D		+	+	+	+		+	+	+		+	+	+	+	+		+		+	+		+		+		+			+		+	+
E		+	+	+					+		+	+	+		+	+	+	+		+		+		+	+		+		+		+	
F		+			+	+			+									+		+	+	+		+			+			+	+	
G		+	+	+			+	+	+	+			+	+			+	+			+	+					+	+		+	+	+
H		+		+		+	+	+	+	+		+	+	+	+	+	+	+	+	+	+		+		+		+				+	
Blocks				I								II							III								IV					
Results																																

(Column 1 region: **N O N E**)

Remarks:

1. All main effects are clear of two-factor interactions.
2. When blocks are not used interactions AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, EH, FH and GH are measurable. When blocks are used interactions AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, FH and GH are measurable.
3. All three-factor and higher order interactions are assumed negligible.

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Table 37
8 Treatments and 32 Items
Design: 1/8 X 2^8 Factorial (Ref. 6, page 30)

ITEM NUMBERS

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A			+		+			+	+			+		+				+	+		+			+						+	+	
B		+	+		+			+		+	+					+		+	+			+	+	+		+	+					
C		+	+		+			+	+		+				+		+					+			+	+	+	+				+
D		+	+	+				+				+	+			+	+								+							+
E									+	+				+		+	+		+	+	+		+	+	+	+	+			+	+	+
F		+				+	+	+				+	+			+		+			+											
G		+	+			+	+		+			+	+				+	+	+		+	+	+		+	+		+		+		
H			+	+	+		+	+	+	+	+	+				+	+	+				+			+					+		
Blocks	I			II			III			IV			V			VI			VII			VIII											
Results																																

Remarks:

1. All main effects are clear of two-factor interactions.
2. Only the following interactions are measurable: AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, EH, FH and GH whether or not blocks are used.
3. All three-factor and higher order interactions are assumed negligible.

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Table 38
9 Treatments and 32 Items
Design: 1/16 X 2⁹ Factorial (Ref. 6, page 43)

ITEM NUMBERS

Treatments: A, B, C, D, E, F, G, H, I (NONE)

Blocks: I, II

Results

Remarks:

1. All main effects are clear of two-factor interactions.

2. Only the following interactions are measurable: AH, AI, BH, BI, CH, CI, DH, DI, EH, EI, FH, FI, GH, GI and HI whether or not blocks are used.

3. All three-factor and higher order interactions are assumed negligible.

Table 3)
2 Treatments and 32 Items
Design: 1/16 X 2⁹ Factorial (Ref. 6, Page 42)

$$\text{Design: } 1/16 \times 2^9 \text{ Factorial (Ref. 6, Page 42)}$$

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+			+	+		+			+	+	+		+			+	+		+	+		+			+	+			+
B		+	+			+	+	+	+	+			+	+	+						+	+	+	+	+	+	+	+		+	+	
C		+	+			+	+	+			+	+				+	+	+	+	+	+			+		+	+			+	+	
D		+	+			+	+	+	+	+	+	+	+	+	+	+																
E		+	+	+	+	+				+	+	+	+	+	+	+			+	+	+			+	+	+			+			+
F			+	+	+	+					+	+	+	+	+		+	+	+	+	+	+		+	+		+		+	+	+	+
G			+	+	+	+	+		+		+	+	+	+				+	+	+	+	+	+	+				+	+			
H			+	+	+			+	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+	+	+
I			+	+					+	+	+		+	+	+	+	+	+			+	+	+	+	+	+	+		+	+	+	+
Blocks	I								II								III								IV							
Results																																

(NONE)

Remarks:
1. All main effects are clear of two-factor interactions.
2. Only the following interactions are measurable: AH, AI, BH, BI, CH, CI, DH, DI, EH, EI, FH, FI, GH, GI and HI whether or not blocks are used.
3. All three-factor and higher order interactions are assumed negligible.

APPENDIX 4

Table 40
9 Treatments and 32 Items
Design: 1/16 X 2⁹ Factorial (Ref. 6, page 42)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+		+			+		+	+		+			+	+			+			+		+			+		+	+	
B		+	+			+	+			+	+			+	+		+			+			+		+		+		+			+
C		+	+		+			+	+			+		+	+		+			+	+								+			+
NONE																																
D		+			+	+	+	+	+			+				+		+	+	+			+	+		+	+			+	+	
E				+						+	+		+	+				+	+			+	+			+						
F		+			+	+	+		+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+		+
G		+		+		+	+		+												+		+	+	+	+		+	+	+	+	+
H		+		+		+	+	+	+	+	+	+		+	+	+	+	+		+			+	+	+		+		+		+	+
I				+	+	+	+	+		+	+	+	+		+	+	+			+			+	+			+	+	+		+	+
Blocks	I				II				III				IV				V				VI				VII				VIII			
Results																																

Remarks:
1. All main effects are clear of two-factor interactions.
2. Only the following interactions are measurable: AH, AI, BH, BI, CH, CI, DH, DI, EH, EI, FH, FI, GH, GI and HI whether or not blocks are used.
3. All three-factor and higher order interactions are assumed negligible.

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APPENDIX 4

Table 41
10 Treatments and 32 Items
Design: 1/32 X 2^10 Factorial (Ref. 6, page 53)

(Factorial design matrix — Treatments A through J across 32 items, divided into Blocks I and II, with rows for Blocks and Results. Column headings numbered 1–32; "TWO BLOCKS" indicated across the top; "NONE" marked vertically within Block I.)

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

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APPENDIX 4

Table 42
10 Treatments and 32 Items
Design: 1/32 X 2¹⁰ Factorial (Ref. 6, page 52)

ITEM NUMBERS

Treatments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+		+		+		+	+		+		+		+		+	+		+		+		+	+		+		+		+	
B		+		+	+	+		+	+		+		+		+		+		+		+		+			+		+		+		+
C		+		+	+		+		+		+		+	+	+	+	+		+		+			+		+	+	+	+	+		
D		+		+	+		+		+		+			+		+		+				+	+		+					+	+	+
E		+		+	+		+			+		+			+	+			+			+		+	+		+			+		+
F		+		+	+		+			+		+	+							+	+		+	+		+	+				+	
G		+	+			+	+			+	+		+	+	+		+			+	+		+	+	+			+	+	+	+	+
H		+	+	+		+	+			+	+	+	+	+	+	+		+		+	+	+				+		+	+	+	+	
I			+	+			+	+			+	+		+	+	+	+	+	+			+		+	+	+			+	+	+	
J			+	+	+		+	+	+		+	+			+	+	+		+				+	+	+		+	+			+	+
Blocks			I									II							III								IV					
Results																																

(Column 1: NONE)

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

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Table 43
10 Treatments and 32 Items
Design: $1/32 \times 2^{10}$ Factorial (Ref. 6, page 52)

ITEM NUMBERS

Treat- ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+		+		+		+			+		+		+		+	+		+		+		+	+		+		+	+		+
B		+		+	+	+	+	+	+		+		+		+			+	+	+	+	+	+	+		+	+	+	+	+	+	
C		+	+	+	+		+		+	+	+				+	+	+	+	+	+	+		+	+		+			+		+	
D		+		+	+		+		+		+		+	+						+	+	+				+	+		+	+		+
E		+		+	+		+					+	+	+	+		+	+		+		+	+	+	+		+	+	+	+		+
F		+	+	+	+		+					+	+		+		+	+											+			
G		+	+			+	+		+	+	+		+		+		+	+	+	+	+	+	+	+	+	+		+	+		+	+
H		+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+	+		+	+			+	+	
I		+	+	+			+	+	+		+	+		+	+		+				+					+				+	+	
J			+	+	+		+	+			+	+			+	+		+	+	+		+	+	+	+	+	+	+	+		+	+

(The treatments F, G region carries the label "NONE")

Blocks	I				II				III				IV				V				VI				VII				VIII			
Results																																

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

APPENDIX 4

Table 44
11 Treatments and 32 Items
Design: 1/64 X 2¹¹ Factorial (Ref. 6, page 58)

Treat-ments	ITEM NUMBERS																															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+			+	+			+	+		+	+	+		+	+	+			+	+			+	+			+	+	
B		+	+			+	+			+	+			+	+				+	+	+	+	+					+				+
C		+	+	+		+	+		+		+		+				+		+	+	+		+	+		+		+	+	+	+	
D	+	+	+	+			+		+			+	+			+							+		+			+	+			+
E			+	+	+		+	+		+		+	+	+		+	+		+	+			+	+	+	+			+	+		+
F			+	+	+			+			+	+			+	+		+	+	+			+	+	+	+	+	+		+	+	+
G			+	+	+	+		+			+	+			+	+		+	+	+			+		+	+	+		+	+		
H			+	+	+	+	+				+	+		+				+	+	+	+	+		+				+	+	+	+	
I			+	+	+	+	+	+	+	+	+	+	+	+		+	+			+	+		+	+	+	+					+	+
J		+	+	+	+	+			+	+	+	+	+			+				+		+	+		+	+					+	+
K	+	+	+	+	+	+	+	+	+	+	+		+			+	+		+	+	+	+	+		+	+		+		+	+	
Blocks				I								II								III								IV				
Results																																

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

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Table 45
12 Treatments and 32 Items
Design: 1/128 X 2^{12} Factorial (Ref. 6, page 65)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+		+	+		+			+	+	+	+		+			+		+	+	+	+			+		+	+		+	
B		+	+			+	+			+	+		+	+	+		+	+	+			+	+		+	+	+	+	+		+	
C		+			+		+	+		+					+	+		+	+		+		+	+			+	+		+		+
D			+	+	+	+	+	+			+	+	+		+	+	+	+	+	+		+	+	+	+	+			+		+	
E				+	+	+	+	+					+	+	+	+	+	+	+	+	+	+	+	+	+		+		+	+		+
F							+	+	+					+	+	+	+	+	+		+	+	+	+	+	+		+	+	+	+	+
G						+	+	+	+	+	+	+	+	+		+	+		+	+	+		+						+	+		
H			+	+	+	+	+	+	+	+		+	+	+			+	+	+	+			+	+	+	+			+	+		
I							+	+	+		+	+				+	+	+	+	+	+	+	+				+		+	+		
J		+			+	+	+		+		+		+	+		+		+	+	+	+	+	+					+	+	+		+
K		+	+			+	+				+	+	+			+	+	+					+		+	+		+	+			
L		+	+		+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+		+	+	+	+		+	+	+	+	+
Blocks	I								II								III								IV								
Results																																

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

Table 46
13 Treatments and 32 Items
Design: $1/256 \times 2^{13}$ Factorial (Ref. 6, page 74)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+	+		+	+	+			+	+			+	+			+	+	+		+	+	+		+	+			+	+	
B		+		+		+		+		+		+		+		+		+			+	+		+		+		+		+		+
C		+		+	+		+			+	+	+	+			+		+		+	+		+			+		+	+	+	+	
D		+	+		+			+		+		+	+	+		+		+	+	+	+		+	+		+	+					+
E		+		+	+	+		+	+	+	+			+		+			+		+		+			+	+	+	+			
F		+		+	+		+	+	+		+					+	+		+	+	+		+		+		+					+
G		+		+	+	+			+		+			+		+	+		+			+		+				+	+	+	+	+
H		+		+	+	+	+			+			+		+	+	+	+	+	+							+			+		
I		+	+	+		+	+	+	+			+	+			+	+			+	+			+		+	+			+	+	
J		+		+		+		+	+				+		+		+	+		+	+	+		+				+		+	+	+
K		+		+		+	+	+	+	+	+		+		+				+	+		+		+			+	+	+			
L		+	+				+		+		+				+		+	+	+		+			+	+							
M		+	+		+				+			+																				
Blocks					I								II								III								IV			
Results																																

Remarks:
1. All main effects are clear of two-factor interactions.
2. None of the two-factor interactions are measurable.
3. All three-factor and higher order interactions are assumed negligible.

Table 47

31 Treatments and 32 Items
Design: Multifactorial (Ref. 5, page 323)

ITEM NUMBERS

Treat-ments	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A		+			+		+	+			+	+	+	+	+				+	+		+	+	+	+	+		+				+
B		+	+	+		+	+			+	+	+	+	+		+	+	+	+		+	+	+	+			+				+	
C			+		+	+			+	+	+	+	+		+	+	+	+		+	+	+	+	+	+					+	+	+
D		+	+	+	+			+	+	+	+	+			+	+	+	+	+	+	+	+		+	+				+	+		+
E			+	+	+	+	+	+	+	+	+			+	+	+	+	+	+		+		+					+			+	
F		+	+	+	+	+	+	+		+			+	+	+	+	+	+		+	+	+	+				+			+	+	+
G		+	+	+		+	+	+	+				+	+		+	+	+			+					+			+	+		+
H		+		+	+	+	+		+					+	+	+			+	+		+			+			+			+	+
I					+	+	+		+				+		+		+			+	+			+		+	+			+	+	+
J			+	+	+	+	+		+	+		+	+	+	+	+	+	+					+		+			+	+	+	+	+
K		+	+	+	+	+	+	+	+	+	+	+	+		+	+						+						+		+	+	+
L		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+						+	+	+	+	+	+	+	+	+	+	+
M		+	+	+	+	+	+	+	+	+	+	+	+	+				+				+		+	+	+	+	+	+	+	+	+
N				+	+	+	+	+	+	+				+		+		+												+	+	+
O			+	+	+	+	+			+	+	+	+		+	+	+	+		+	+		+									
P		+	+	+	+	+	+	+	+		+	+					+		+	+	+	+	+	+	+	+	+	+	+	+	+	+
Q			+	+	+	+	+	+	+	+	+		+	+	+			+		+	+	+	+	+	+	+	+	+	+	+	+	+
R		+	+	+	+	+	+		+						+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
S		+	+		+	+	+		+				+												+	+	+	+	+	+	+	+
T		+	+	+	+			+														+		+	+	+	+	+	+	+	+	+
U		+	+	+	+	+	+	+	+	+								+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
V		+	+	+	+	+	+		+				+	+	+	+	+	+	+	+	+	+			+	+	+	+	+	+	+	
W			+	+	+	+				+				+	+						+	+										
X		+	+	+	+		+		+					+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Y		+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
21		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+
22		+	+					+		+	+	+	+	+	+	+	+	+					+	+	+	+	+	+	+	+	+	
23			+						+	+													+	+	+	+	+			+	+	+
24		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
25																																	
26			+			+			+	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	

(NONE appears in the blank leading column of the table.)

Results:

Remarks:

1. All main effects are confused with interactions.
2. All interactions are assumed negligible.
3. The treatment combinations in the individual rows of this design can be used in any combination of two or more up to and including 31 treatments.

APPENDIX 5

REFERENCES

1. Lusser, Robert, *Unreliability of Electronics — Cause and Cure*, Redstone Arsenal, Huntsville, Ala., Nov. 1957.

2. Lusser, Robert, *Reliability Through Safety Margins.* Redstone Arsenal, Huntsville, Ala., Oct. 1958.

3. Dixon and Massey, *Introduction to Statistical Analysis*, McGraw-Hill Book Co., NYC, 1957, Second Edition.

4. Davies, O.L. (Editor), *Statistical Methods in Research and Production,* Hafner Publishing Co., NYC, 1957, Third Edition.

5. Plackett and Burman, *The Design of Optimum Multifactorial Experiments,* Biometrika, Vol 33, 1946, page 305.

6. *Fractional Factorial Experiments Designs for Factors at Two Levels,* NBS Applied Math. Series No. 48.

7. Fisher, R.A., *Statistical Methods for Research Workers,* Hafner Publishing Co., NYC, 1950.

8. Mainland, Herrera, and Sutcliffe, *Tables for Use With Binomial Samples,* Department of Medical Statistics, NY University College of Medicine, NYC, 1956.

9. Pearson, E.S., and Hartley, H.O., *Biometrika Tables for Statisticians,* Volume 1, Cambridge University Press, Cambridge, England, First Edition, 1956.

10. Churchman, C.W., *Tables for Sensitivity Tests Conducted at Two Stimuli,* Frankford Arsenal Memorandum Report No. MR-540, May 1953.

11. Bulfinch, A. *Characteristics of Various Methods of Collecting Data in Tests of Increased Severity,* Picatinny Arsenal Technical Report No. 2627.

12. Bulfinch, A. *Design of Experiments,* Picatinny Arsenal Report No. ORDBB-NR-107, 1960.

13. Bulfinch, A., *Safety Margins and Ultimate Reliabil y*, Picatinny Arsenal Report No. ORDBB-NR-106, 1960.

14. Hurst, D.C. and Duffett, J.R., *Statistical Methodology for the Analysis and Evaluation of Defect Data on the Redstone Missile System* Virginia Polytechnic Institute Technical Report No. 4, Part A, Nov. 1958, Part C, April 1959.

15. Franktik, R.O., *The Determinations, Applications, and Limitations of Circuit Reliability Equations,* Sandia Corp. Division 5131, April 26, 1954.

16. DeCicco, Henry, *The reliability of weapons Systems Estimated from Component Test Data Alone,* Ordnance Special Weapons — Ammunition Command, 1 Dec. 59.

17. Orkand, D.S. *A Monte Carlo Method for Determining Lower Confidence Limits for System Reliability on the Basis of Sample Component Data.* Picatinny Arsenal, June 1960.

18. Davies, O.L. (editor), *Design and Analysis of Industrial Experiments,* Hafner Publishing Co., N.Y.C., 1954.

19. Scheffe, Henry, *The Analysis of Variance,* John Wiley and Sons, Inc., N.Y.C., 1959.

20. Odell, P.L., *Tables and Graphs for Determining an Upper Confidence Bound on the Number of Defectives in a Finite Population,* NAVORD Report 7123, U.S. Naval Nuclear Ordnance Evaluation Unit, Albuquerque, N.M., 15 July 1960.

21. Bennett, C.A. and Franklin, N.L., *Statistical Analysis in Chemistry and the Chemical Industry,* John Wiley and Sons, Inc., N.Y.C., 1954.

22. Snedecor, G.W., *Statistical Methods,* Iowa State College Press, Ames, Iowa, Fifth Edition, 1956.

23. Bowker, A.H. and Lieberman, G.J., *Engineering Statistics,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1959.

PREFACE

This handbook is intended as a guide for determining reliability of functioning characteristics of weapon components by testing to failure.

Component reliability of weapon systems is basically a function of engineering design. Margins of safety used in engineering design to create high reliabilities must be measured by testing to failure techniques to obtain unbiased estimates of reliability.

The author does not hold that the concepts and principles presented herein are final. Revisions will inevitably be made as the state of the art advances.

ACKNOWLEDGEMENTS

The following tables have been taken from Government documents:

ppendix 3G

Churchman, C.W., "Statistical Manual: Methods of Making Experimental Inferences", ankford Arsenal, Philadelphia, Pa., Table I: Areas and Their Standard Sampling Errors for the ormal Curve, pages 152 thru 155.

ppendix 3H

Odell, P.L., "Tables and Graphs for Determining an Upper Confidence Bound on the Number Defectives in a Finite Population", U.S. Naval Nuclear Ordnance Evaluation Unit, Albuquerque, .M., Table I, page 9; Table 2, page 10; Table 4, page 14; Table 9, page 33; Table II, page 43; able 14, page 62; Table 17, page 85; Table 19, page 103; and Table 24, page 156.

ppendix 4 (Fractional Factorials

U.S. Dept of Commerce, National Bureau of Standards, "Fractional Factorial Experiment esigns for Factors at Two Levels", Applied Mathematics Series 48, Superintendent of Documents, .S. Printing Office, Washington 25, D.C.

END